國立中正大學101學年度學士班二年級轉學生招生考試試題

學系別：財務金融學系 科目：微積分

第 1 節

1. The demand function for a certain commodity is $p = -0.01x^2 - 0.15x + 6$, where p is the wholesale unit price in dollars and x is the quantity demanded each week, measured in units of a thousand. (a) Let $R(x)$ be the revenue function. Find x such that the marginal function $MR(x)$ is zero and the corresponding price. (b) Compute the elasticity of the demand (which measures the effect that a percentage change in price has on the percentage change in the quantity demanded i.e. $E(p) = -\frac{px'(p)}{x(p)}$.) and the unit price when $x = 5$. (10%)

2. Suppose the price (in dollars per ton) for oat bran is $D(q) = 149 - 2e^q$ when the demand for the product is q tons. Also, suppose the function $S(q) = e^q - 1$ gives the price(in dollars per ton) when the supply is q tons. Find the consumers' surplus and the producers' surplus. (8%)

3. Mary makes an investment that is expected to generate \$500/month for the next 5 years. If the income is reinvested in a business earning interest at the rate of 5%/year compounded continuously. Using a definite integral find the total accumulated value of this income stream at the end of 10 years. (7%)

4. Determine the critical points of the function $f(x,y) = 3x^{2/3}y^{4/3} + 2xy + 2y + y^2$. Then use the second derivative test to classify the nature of each point, if possible. (10%)

5. Use method of Lagrange multiplier to find the maximum value of $f(x,y) = 12x^{1/4}y^{3/4}$ subject to $3x + y = 1200$. Also, find the points at which these extreme values occur. (10%)

6. Suppose x is a normal distribution with mean 57 and standard deviation 3. Use the fourth Taylor polynomial (of degree 4) of $e^{-z^2/2}$ about $z = 0$ to approximate the probability $p(x > 60)$. ($\frac{1}{\sqrt{2\pi}} \approx 0.4$) (10%)

7. Evaluate the integral $\int_0^1 \sqrt{2 - x^2}\, dx$ by interpreting it as the area of some region in the xy-plane. (6%)

8. Evaluate the integral (a) $\int_0^1 \int_{\sqrt{y}}^1 ye^{x^5}\, dx\, dy$, (b) $\int_{-\infty}^{\infty} \frac{x^2}{1+e^{x^3}}\, dx$. and (c) $\int_0^1 x \ln(x^2 + 1)\, dx$. (18%)

9. Find $f_x(0,0)$ and $f_y(0,0)$ if it exists, for $f(x,y) = \begin{cases} \frac{x^2e^{2x}+y^2}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$ (6%)

10. Evaluate the derivative $f'(-1)$, where $f(x) = \int_1^{x^2} t^3\sqrt{1 + 8t^3}\, dt$ (5%)

11. Graph the function $f(x) = 4x^{-1/2} + \frac{1}{3}x^{3/2}$, considering the domain, critical points, regions where the function is increasing or decreasing, points of inflection, regions where the function is concave upward or concave downward, intercepts where possible, and asymptotes where applicable. (10%)

Please write the procedures how you get the answers in detail.