

2025 台灣財務工程學會年會暨國際學術研討會

International Conference for Financial Engineering Association of Taiwan

金融新世代：智能技術與可持續創新
New Generation of Finance-Intelligent Technology and Sustainable Innovation.

| 活動手冊 | 6—13 Fri.



| 主辦單位 |



| 承辦單位 |



國立中正大學
財務金融學系



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2025 年台灣財務工程學會年會暨國際學術研討會

理事長歡迎詞

本次年會暨國際學術研討會由台灣財務工程學會與國立中正大學財務金融學系共同舉辦。台灣財務工程學會成立於 2003 年，創立以來持續以最新財務工程脈絡訊息之傳遞、搭建具影響力之財務工程與金融市場交流平台、以及培育國內財務工程與金融創新相關領域人才為宗旨。每年一度的年會暨國際學術研討會除了為本會的年度盛會外，研討會所產出的學術成果也對促進國內金融產品開發以及交易與結算制度改革等主題扮演重要的角色。此外，本會也因應重大財金議題，以實務論壇為橋樑建立起國內外產官學界的交流平台，與會產官學者的參與付出促使本會得以在提昇國內財務工程之學術發展與實務應用方面逐步發揮關鍵的影響力。本人謹代表主辦單位，向與會的所有貴賓表達最誠摯的謝意與敬意。

本會今年的論文投稿相當踴躍，共計有約 80 篇學術論文會在年會當中發表，主題涵括數位貨幣、綠色金融、機器學習、衍生性金融商品、ESG 等多項前瞻的研究議題，以及資產訂價、公司理財、計量經濟學、以及銀行學等奠基當代財務金融發展的重要課題。在本次年會中，本會特別邀請國際知名學者 University of Hawaii 的 Prof. Ghon Rhee 以及新加坡國立大學經濟學系教授的 Prof. Yi-Chun Chen 擔任大會專題主講人，講述現今財務工程領域前沿的研究主題。對於入選本次研討會之文稿，本會也邀請 Pacific-Basin Finance Journal、中山管理評論、以及人文及社會科學集刊籌辦特刊，盼能透過學術先進的共襄盛舉一同推動國內財務工程領域之學術發展。

本次年會得以順利舉行，得力於許多人全心全力的協助。首先，要特別感謝籌備委員會總幹事林子綾教授，議程委員林丙輝教授、張傳章教授、張森林教授、曾郁仁教授、蔡蒔銓教授、王衍智教授、江彌修教授、林士貴教授、林智勇教授、張眾卓教授、黃泓人教授、黃瑞卿教授、楊曉文教授、以及謝文良教授。此外，也要感謝中正大學財務金融學系所有師生與工作人員、甄選論文與規劃講座的專家學者、所有場次的主持人、學術論文發表人與評論人，以及實務講座主講人。我們更要感謝各位與會者的熱心參與及對學會的支持；您們是研討會得以舉辦成功、學會得以持續茁壯的關鍵。

也特別感謝合辦單位國家科學及技術委員會，贊助單位國立中正大學管理學院、臺灣證券交易所股份有限公司、財團法人中華民國證券櫃檯買賣中心、臺灣期貨交易所股份有限公司、臺灣集中保管結算所股份有限公司、三商美邦人壽保險股份有限公司、資誠教育基金會。有這些單位在財務上無私的支持，才能使今年年會舉辦的如此成功。

最後，敬祝各位先進與貴賓身體健康、萬事如意。

 敬上
台灣財務工程學會理事長

中正大學管理學院院長歡迎信

親愛的貴賓：

謹代表國立中正大學管理學院，誠摯歡迎您蒞臨參加「2025年台灣財務工程學會年會暨國際學術研討會」。

本次年會由台灣財務工程學會主辦、中正大學財務金融學系承辦，並獲得國科會指導，匯聚了來自國內外財務工程與金融科技領域的傑出學者與實務專家，共同探討「金融新世代：智能技術與可持續創新」等前沿議題。在人工智慧、大數據、區塊鏈等技術快速發展的今日，金融產業正迎來轉型契機；而如何結合這些技術推動永續金融發展，更是學術與實務界共同關注的核心課題。

我們相信，本次會議所涵蓋的研究主題，無論是機器學習在投資決策中的應用、ESG因子的量化建模，亦或是綠色金融商品的風險評估，都將促進跨領域的深度對話與合作，為未來金融市場的創新與穩健發展注入新動能。

除了豐富的學術交流外，我們誠摯邀請您在會議之餘，親身體驗嘉義的在地魅力。無論是登上阿里山欣賞日出雲海與森林鐵道的歷史風情，漫步在充滿人文氣息的檜意森活村與故宮南院，還是品嚐噴香的火雞肉飯、砂鍋魚頭與在地小吃，都能為您的旅程增添難忘回憶。嘉義的溫暖人情與悠閒步調，將讓您在緊湊的日程中找到片刻寧靜與愉悅。

中正大學位處嘉義阿里山山腳，擁有得天獨厚的自然環境與靜謐的學術氛圍，非常榮幸能在此迎接各位的到來。希望透過本次會議，您不僅能與同儕深入交流、激盪創新思維，也能享受中正大學與嘉義地區的人文與風景之美。

再次感謝您的參與與支持，祝福本次會議圓滿成功，並祝您旅途平安、收穫豐碩。

敬祝

健康愉快 萬事如意



敬上

中正大學管理學院院長

中華民國 114 年 6 月

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| 候補監事 | 何柏欣 | 國立中央大學財金系教授 |

Conference Agenda

Date: June 13, 2025 (Friday)
 Chung Cheng University

Venue: 1st Floor, College of Management, National

| Time | Conference Program | | | | | |
|---------------------------|---|---|--------------------------------------|--|---|--|
| 08:30 – 09:00 | Registration and Reception / 1st Floor | | | | | |
| 09:00 – 09:05 | <p>[Opening Ceremony] Professor Shaw-Jenq Tsai 蔡少正 / President, Chung Cheng University Professor Robin K. Chou 周冠男 / President, FeAT</p> | | | | | |
| 09:05 – 09:55 | <p>[Keynote Speech] Professor S. Ghon Rhee, University of Hawaii, USA Topic: Natural Disasters and the Real Effect of Skilled Labor Mobility Chair: Professor Robin K. Chou 周冠男 / President, FeAT</p> <p style="text-align: right;">(Room 101)</p> | | | | | |
| Academic Session I | | | | | | |
| 10:05 – 11:05 | SESSION 1-1 ESG and green finance | SESSION 1-2 Corporate finance and governance | SESSION 1-3 ESG and green finance | SESSION 1-4 Cryptocurrency and FX markets | SESSION 1-5 Artificial intelligence and machine learning 中文場次 | SESSION 1-6 Insurance and risk management 中文場次 |
| | Chair: 蔡明宏副研究員 中研院人社中心 | Chair: 何柏欣教授 中央財金 | Chair: 林建甫講座教授 中信金融管理學院 | Chair: 俞明德教授 清大計財 | Chair: 鄭宏文教授 東吳資料 | Chair: 李永琮教授 嘉大財金 |
| | Room 102 | Room 103 | Room 105 | Room 106 | Room 119 | Room 120 |

| Time | Conference Program | | | | | |
|---------------|--|---|--|--|---|---|
| 11:10 – 12:10 | Academic Session II | | | | | [Practical Session] |
| | SESSION 2-1 ESG and green finance | SESSION 2-2 Corporate finance and governance | SESSION 2-3 Quantitative trading and investment | SESSION 2-4 Other topics related to financial markets | SESSION 2-5 Corporate finance and governance 中文場次 | 交大資財博士 曹光志 Topic: Taiwan Dollar Surge and Currency Mismatches in Taiwanese Life Insurers' |
| | Chair: 鄭揚耀教授 中正財金 | Chair: 黃泓人教授 中央財金 | Chair: 陳安行教授 中正財金 | Chair: 王樹鳳教授 Ajou University | Chair: 許和鈞教授 銘傳財金 | Chair: 林智勇教授 交大資財 |
| | Room 102 | Room 103 | Room 105 | Room 106 | Room 119 | Room 120 |
| 12:10 – 13:10 | Luncheon / Atrium | | | FeAT Annual Meeting / Room 101 | | |
| 13:15 – 14:15 | <p data-bbox="958 831 1196 863">[Keynote Speech]</p> <p data-bbox="707 882 1464 914">Professor Yi-Chun Chen, National University of Singapore</p> <p data-bbox="707 933 1464 965">Topic: The Wisdom Of The Crowd And Higher-Order Beliefs</p> <p data-bbox="633 984 1525 1016">Chair: Professor Donald D.H. Lien 連大祥, University of Texas, USA</p> <p data-bbox="1917 1038 2056 1070">(Room 101)</p> | | | | | |

| Time | Conference Program | | | | | |
|----------------------|--------------------------------------|---|---|---|---|--|
| 14:25 – 15:45 | Academic Session III | | | | | |
| | SESSION 3-1 ESG and green finance | SESSION 3-2 Asset pricing: capital market efficiency and anomalies | SESSION 3-3 Behavioral finance and risk management | SESSION 3-4 Artificial intelligence and machine learning | SESSION 3-5 Behavioral finance and risk management 中文場次 | SESSION 3-6 ESG and green finance 中文場次 |
| | Chair: 李修全教授 銘傳財金 | Chair: 曾郁仁教授 台大財金 | Chair: 林士貴教授 政大金融 | Chair: 李韋憲教授 中央財金 | Chair: 董澍琦教授 中興財金 | Chair: 吳仰哲教授 逢甲財金 |
| | Room 102 | Room 103 | Room 105 | Room 106 | Room 119 | Room 120 |
| 15:45 – 16:00 | Coffee Break / Atrium | | | | | |
| 16:00 – 17:20 | Academic Session IV | | | | | |
| | SESSION 4-1 ESG and green finance | SESSION 4-2 Corporate finance and governance | SESSION 4-3 Behavioral finance and risk management | SESSION 4-4 Derivatives and risk management | SESSION 4-5 Other topics related to financial markets | SESSION 4-6 Other topics related to financial markets 中文場次 |
| | Chair: 賴宏彬教授 中正經濟 | Chair: 蔡維哲教授 中山財管 | Chair: 翁培師教授 中山財管 | Chair: 黃瑞卿教授 中央財金 | Chair: 葉錦徽教授 中央財金 | Chair: 賴雨聖教授 暨南財金 |
| | Room 102 | Room 103 | Room 105 | Room 106 | Room 119 | Room 120 |
| 17:30 ~ | Happy Ending | | | | | |

Notes: Each paper presentation is allocated 15 minutes, followed by 5 minutes for discussion per paper.

Keynote Speech

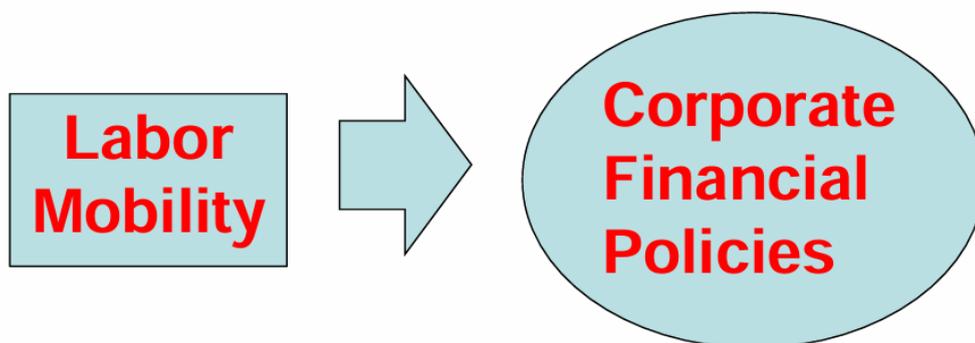
Natural Disasters and Real Effect of Skilled Labor Mobility

Yuna Heo, University of Basel
S. Ghon Rhee, University of Hawaii
June 2025

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Main Focus

Natural Disaster is the key channel



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Natural Disasters

The US Environmental Protection Agency (EPA): **Key Threats** to workers' health caused by climate change

- Heat illnesses
- Respiratory illnesses
- Physical and mental health effects
- Insect-related diseases
- Pesticide-related effect

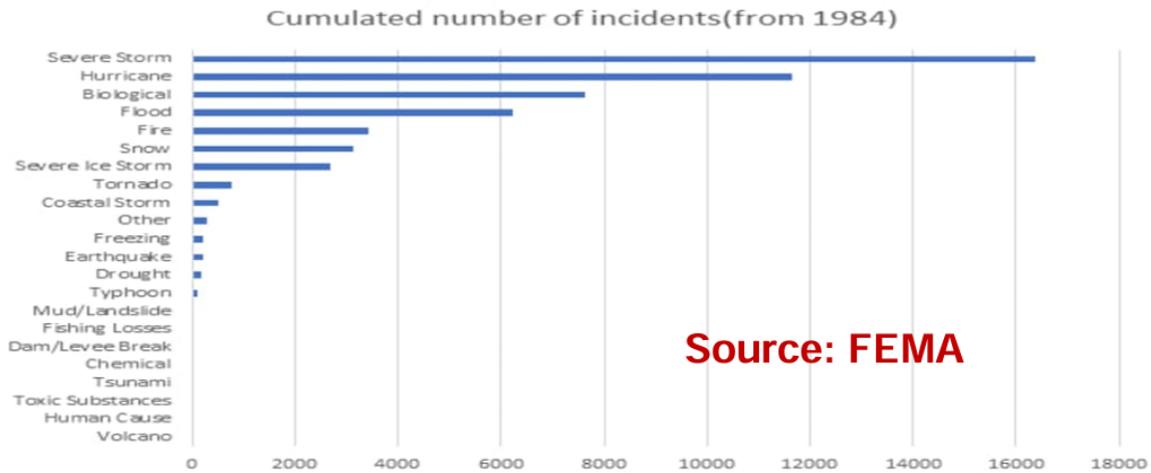
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Natural Disasters

- **Sources of Climate Disasters:**
Federal Emergency Management Agency (FEMA)
- **FEMA provides the data for 61,344 disaster declarations since 1953**

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US Natural Disasters (from 1984)



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Labor Mobility (I)

Two Types of Labor

- **Skilled Labor Mobility**
- **Routine-Task Labor (or Unskilled Labor Mobility)**

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Skilled Labor Mobility (I)

- **Belo, Li, Lin, and Zhao (RFS, 2017):** A negative relation between firms' hiring rates and future stock returns. This relation is **steeper** when skilled workers are involved
- **Qiu and Wang (RCFS, 2021):** Measure Skilled Labor Risk using 10-K reports using the textual analysis

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Skilled Labor Mobility (II)

- **Extreme Climate Events (hurricanes, wildfires, heatwaves, etc.)** affect the labor market
- **Climate Change: Far-Reaching Impacts on Labor Mobility):** Acharya, Bhadwaj, and Tomunen (2023, NBER WP) and Albert, Bustos, and Ponticelli (2023, NBER WP).

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Skilled Labor Mobility (III)

- Firms cannot meet labor demand **without incurring costs**
- The labor adjustment cost **increases with the level of workers' skill**
(Ghaly, Dang, and Stathopoulos, RFS, 2017)

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Skilled Labor Mobility (IV)

- Intuitively, the predicted response of firms exposed to severe natural disasters is to have **extra liquidity buffers....Cash Holdings as Buffer**
- The effect is more crucial if firms have a high share of **skilled workers**
- The skilled labor mobility and climate disasters also have an impact on
 - Firm productivity
 - Product market share
 - Stock Performance
 - Firm growth

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Main Purpose

- We study the real effect of **Skilled Labor Mobility** on Corp Cash Holdings as a response to climate disasters
- **Unskilled Labor Mobility (Routine-Task Labor)** is also examined

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Empirical Questions

Does skilled labor exposed to severe climate disasters affect

- Corporate cash holding policies
- Product market outcomes?
- If so, does state-level climate adaptation policy matter?

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Skilled Labor (I)

Measuring skilled labor at the **firm-level** is challenging

- To proxy the firm-level skilled labor, we rely on the measure developed by Qiu and Wang (RCFS, 2021) using textual analysis

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Skilled Labor (II)

- The Qiu and Wang measure is based on firms' discussions on risk related to skilled labor in their Form 10-K filings
- The benefit of this proxy is that substantial variations exist over-time **at the firm-level**

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Routine-Task Labor (I)

- To proxy unskilled, routine-task labor, the share of routine-task labor (RShare) developed by Zhang (2019, JF)
- Zhang: Firms with a high share of routine-task labor invest more in machines and have lower expected returns
- The data is from the Occupational Employment Statistics (OES) program of the Bureau of Labor Statistics (BLS)

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Routine-Task Labor (II)

- **RShare (a.k.a labor-technology substitution or robotization)** is defined as the ratio of
Total wages paid to its routine-task labor
relative to its total wage expense
- The methodology in developing this measure is based on a commonly used procedure in the labor economics literature (Autor and Dorn, AER, 2013; Bates, Du, and Wang, MS, 2024)

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Data

- Study Period: 1996-2014
- Compustat: Firms except Financials and Utilities
- FEMA Cumulated Number of Natural Disasters
- Skilled Labor: Qiu and Wang (RCFS, 2021)
- Routine Task Labor: Zhang (JF, 2019)

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Summary Statistics

| Variables | MEAN | SD | Median |
|--|-------|-------|--------|
| Cash holdings/TA | 0.220 | 0.244 | 0.120 |
| Sales growth | 0.218 | 0.816 | 0.085 |
| Climate disasters (count) by each state and year | 38.27 | 64.97 | 17.00 |
| Climate disasters (each state and year/total disasters by year | 0.021 | 0.039 | 0.005 |
| Skilled labor mobility (Qiu & Wang, 2021) | 3.636 | 3.605 | 3.000 |
| Routine-task labor (%) (Zhang, 2019) | 0.156 | 0.097 | 0.126 |

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Correlation Matrix

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | |
|------------------------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|-------|
| (1) Cash holdings | 1.000 | | | | | | | | | | | | |
| (2) Climate disasters (count) | -0.065 | 1.000 | | | | | | | | | | | |
| (3) Climate disasters (ratio) | -0.082 | 0.817 | 1.000 | | | | | | | | | | |
| (4) Skilled labor mobility | 0.311 | -0.024 | -0.058 | 1.000 | | | | | | | | | |
| (5) Routine-task labor (%) | -0.249 | -0.011 | 0.012 | -0.266 | 1.000 | | | | | | | | |
| (6) Ln(size) | -0.235 | 0.021 | 0.016 | 0.038 | 0.096 | 1.000 | | | | | | | |
| (7) Market-to-Book | 0.123 | -0.013 | -0.020 | 0.057 | -0.036 | 0.034 | 1.000 | | | | | | |
| (8) Leverage | -0.133 | 0.004 | 0.006 | -0.055 | 0.072 | 0.133 | 0.375 | 1.000 | | | | | |
| (9) Cash Flow | -0.156 | 0.013 | 0.017 | -0.020 | 0.094 | 0.505 | 0.120 | 0.088 | 1.000 | | | | |
| (10) Net Working Capital | -0.101 | 0.007 | 0.012 | -0.025 | 0.114 | 0.398 | 0.158 | 0.081 | 0.744 | 1.000 | | | |
| (11) R&D | 0.382 | -0.046 | -0.053 | 0.136 | -0.154 | -0.426 | 0.010 | -0.107 | -0.619 | -0.440 | 1.000 | | |
| (12) Capital expenditure | -0.184 | 0.018 | 0.050 | -0.076 | 0.168 | 0.032 | 0.011 | 0.019 | -0.011 | -0.006 | -0.024 | 1.000 | |
| (13) Industry cash flow volatility | 0.222 | -0.047 | -0.072 | 0.227 | -0.155 | -0.057 | -0.012 | -0.050 | -0.161 | -0.136 | 0.231 | -0.120 | 1.000 |

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Empirical Results (Table 2, Panel A)

Panel A: Climate disasters, skilled labor mobility, and routine-task labor

| | Liquidity buffers | |
|---|------------------------|------------------------|
| | (1) | (2) |
| Frequent climate disasters | 0.0078** (2.05) | 0.0267*** (4.99) |
| Skilled labor mobility * Frequent climate disasters | 0.0016* (1.92) | |
| Skilled labor mobility | 0.0119*** (13.15) | |
| Routine-task labor * Frequent climate disasters | | -0.0839*** (-3.78) |
| Routine-task labor | | -0.0036 (-0.11) |
| Ln(size) _{i,t} | -0.0104*** (-8.11) | -0.0079*** (-7.78) |
| Market-to-Book _{i,t} | 0.0040*** (15.03) | 0.0041*** (19.96) |
| Leverage _{i,t} | -0.0115*** (-15.89) | -0.0125*** (-22.78) |
| Cash flow _{i,t} | -0.0072** (-2.04) | -0.0124*** (-4.80) |
| Net working capital _{i,t} | 0.0017 (0.38) | 0.0085*** (2.60) |
| Industry cash flow volatility _{i,t} | 0.0060 (0.08) | 0.0001*** (4.20) |
| Acquisitions _{i,t} | -0.0001*** (-11.40) | -0.0002*** (-16.68) |
| Dividend _{i,t} | -0.0302*** (-6.64) | -0.0397*** (-10.66) |
| Year fixed effect | Yes | Yes |
| Industry fixed effect | Yes | Yes |
| Clustered by firm | Yes | Yes |
| Observations | 42464 | 80505 |
| Adjusted R ² | 0.3680 | 0.3453 |

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Empirical Results (Table 2, Panel B)

Panel B: The effect of financial constraints

| | Liquidity buffers | | | |
|--|----------------------|-----------------------|--------------------|-----------------------|
| | (1) | (2) | (3) | (4) |
| <i>Skilled labor mobility</i> | 0.0133*** (18.45) | 0.0133*** (18.51) | | |
| <i>Skilled labor mobility * Frequent climate disasters * Financial constraints</i> | 0.0022*** (3.25) | 0.0044*** (4.86) | | |
| <i>Financial constraints</i> | | -0.0205*** (-5.20) | | -0.0103*** (-3.71) |
| <i>Routine-task labor</i> | | | -0.0421 (-1.39) | -0.0410 (-1.36) |
| <i>Routine-task labor * Frequent climate disasters * Financial constraints</i> | | | 0.0355** (2.12) | 0.0766*** (3.68) |
| Baseline controls | Yes | Yes | Yes | Yes |
| Year fixed effect | Yes | Yes | Yes | Yes |
| Industry fixed effect | Yes | Yes | Yes | Yes |
| Observations | 40799 | 40799 | 76662 | 76662 |
| Adjusted R ² | 0.359 | 0.359 | 0.323 | 0.324 |

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Product Market Outcomes (Table 3)

Product market outcomes_{it} =

$\alpha + \beta_1 * \text{Skilled labor mobility}_{i,t} \text{ (or Routine - task labor}_{i,t})$

$+ \beta_2 * \text{Skilled labor mobility}_{i,t} \text{ (or Routine - task labor}_{i,t})$

$+ \text{frequent natural disasters}_{s,t}$

$+ \gamma * X_{i,t-1} + \epsilon$ (5)

| | Skilled labor mobility | | | Routine-task labor | | |
|--|------------------------|----------------------|-----------------------|---------------------|--------------------|--------------------|
| | Sales growth (1) | MSG_SIC (2) | MSG_FF (3) | Sales growth (4) | MSG_SIC (5) | MSG_FF (6) |
| <i>Skilled labor mobility * Frequent climate disasters</i> | -0.0061*** (-2.87) | -0.0054** (-2.55) | -0.0056*** (-2.64) | | | |
| <i>Skilled labor mobility</i> | 0.0231*** (10.03) | 0.0220*** (9.70) | 0.0230*** (10.03) | | | |
| <i>Routine-task labor * Frequent climate disasters</i> | | | | -0.0272 (-0.96) | -0.0295 (-1.06) | -0.0260 (-0.92) |
| <i>Routine-task labor</i> | | | | -0.0117 (-0.16) | -0.0722 (-1.03) | -0.0230 (-0.32) |
| Controls | Yes | Yes | Yes | Yes | Yes | Yes |
| Year fixed effect | Yes | Yes | Yes | Yes | Yes | Yes |
| Industry fixed effect | Yes | Yes | Yes | Yes | Yes | Yes |
| Clustered by firm | Yes | Yes | Yes | Yes | Yes | Yes |
| Observations | 40944 | 40944 | 40944 | 77097 | 77097 | 77097 |
| Adjusted R ² | 0.0568 | 0.0457 | 0.0468 | 0.0497 | 0.0401 | 0.0407 |

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Alternative Skilled Labor

- Belo, Li, Lin, and Zhao (2017, RFS) construct an industry-specific labor skill index from the Bureau of Labor Statistics (BLS)
- To determine the number of high (low) skill workers in **each industry**, they use data on the number of workers by occupation in each industry from the BLS Occupational Employment Statistics (OES)

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Using Alternative Measure

| | Liquidity buffers | Product market outcomes | | | Product market differentiation |
|---|-------------------|-------------------------|------------|------------|--------------------------------|
| | | Sales growth | MSG_SIC | MSG_FF | |
| | (1) | (2) | (3) | (4) | (5) |
| Skilled labor (industry-level) * Frequent climate disasters | 0.1459*** | -0.0529*** | -0.0617*** | -0.0628*** | -0.1716*** |
| (Belo et al. 2017) | (14.61) | (-2.63) | (-3.14) | (-3.15) | (-9.94) |
| Controls | Yes | Yes | Yes | Yes | Yes |
| Year fixed effects | Yes | Yes | Yes | Yes | Yes |
| Clustered by firm | Yes | Yes | Yes | Yes | Yes |
| Observations | 34337 | 32779 | 32779 | 32779 | 23457 |
| Adjusted R ² | 0.1773 | 0.0324 | 0.0273 | 0.0271 | 0.2131 |

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State-Level Adaptation Policy

Main Purpose: Make the State Resilient to Climate Shocks

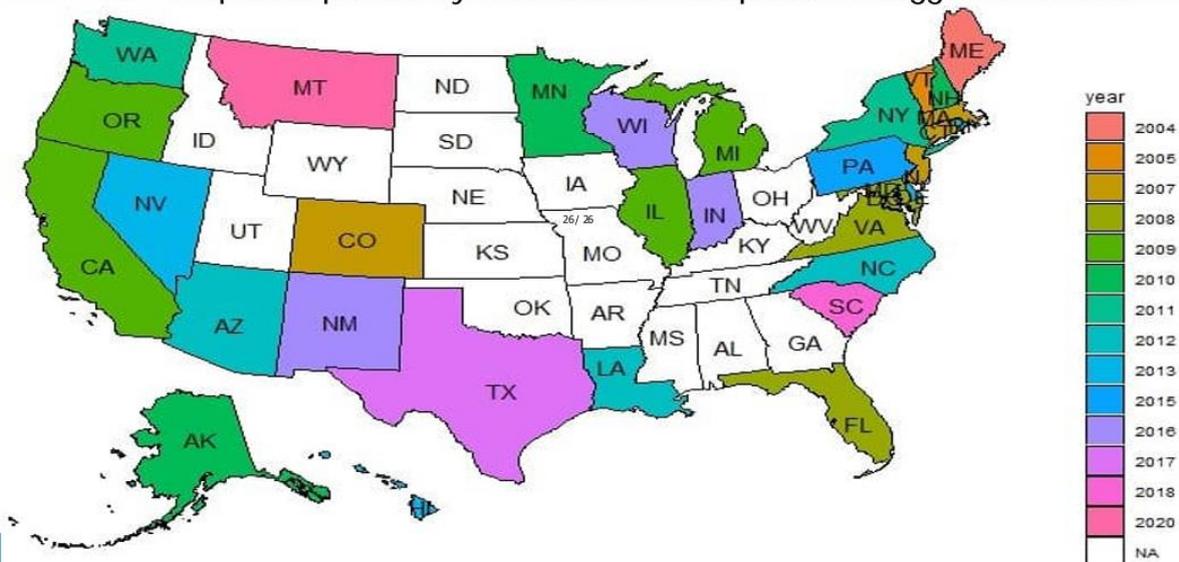
- **20 States finalized Adaptation Policy**
- **5 States have the Adaptation Plans under way**

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State-Level Climate Adaptation Policy as of March 2025

The **climate adaptation policy** aims to prepare for the impacts of climate change and to make states more resilient to adverse effect of climate change. The adaptation policies by each state are adopted in a staggered manner.



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Summary Statistics by State (1)

| State | Number of firms | Average of cash holdings | Average of climate exposure |
|----------------------|-----------------|--------------------------|-----------------------------|
| Alabama | 100 | 0.16 | 0.28 |
| Alaska | 32 | 0.05 | 0.67 |
| Arizona | 331 | 0.27 | 1.24 |
| California | 5431 | 0.39 | 0.58 |
| Colorado | 465 | 0.22 | 1.15 |
| Connecticut | 425 | 0.18 | 1.19 |
| District of Columbia | 72 | 0.31 | 1.16 |
| Delaware | 83 | 0.31 | 0.68 |
| Florida | 606 | 0.17 | 0.39 |
| Georgia | 574 | 0.19 | 0.76 |
| Hawaii | 3 | 0.05 | 4.84 |
| Iowa | 91 | 0.12 | 0.65 |
| Idaho | 42 | 0.13 | 0.31 |
| Illinois | 921 | 0.17 | 0.84 |
| Indiana | 244 | 0.17 | 0.88 |
| Kansas | 106 | 0.16 | 0.26 |
| Kentucky | 129 | 0.10 | 0.21 |
| Louisiana | 43 | 0.03 | 0.74 |
| Massachusetts | 1851 | 0.40 | 0.71 |
| Maryland | 363 | 0.33 | 0.66 |
| Maine | 14 | 0.21 | 0.19 |
| Michigan | 405 | 0.16 | 1.21 |
| Minnesota | 668 | 0.20 | 0.65 |
| Missouri | 333 | 0.14 | 0.88 |

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Summary Statistics by State (2)

| State | Number of firms | Average of cash holdings | Average of climate exposure |
|----------------|-----------------|--------------------------|-----------------------------|
| Mississippi | 5 | 0.02 | 0.37 |
| Montana | 17 | 0.26 | 0.37 |
| North Carolina | 444 | 0.21 | 0.71 |
| North Dakota | 10 | 0.09 | 0.65 |
| Nebraska | 72 | 0.22 | 1.28 |
| New Hampshire | 112 | 0.17 | 0.50 |
| New Jersey | 953 | 0.29 | 0.56 |
| Nevada | 171 | 0.17 | 0.52 |
| New York | 1358 | 0.24 | 0.61 |
| Ohio | 693 | 0.12 | 0.86 |
| Oklahoma | 42 | 0.06 | 0.64 |
| Oregon | 189 | 0.25 | 0.49 |
| Pennsylvania | 920 | 0.19 | 0.64 |
| Rhode Island | 62 | 0.22 | 0.54 |
| South Carolina | 117 | 0.20 | 0.41 |
| South Dakota | 41 | 0.13 | 0.49 |
| Tennessee | 322 | 0.10 | 0.54 |
| Texas | 1496 | 0.19 | 0.76 |
| Utah | 206 | 0.29 | 0.62 |
| Virginia | 488 | 0.19 | 0.69 |
| Vermont | 1 | 0.01 | 0.00 |
| Washington | 482 | 0.39 | 0.63 |
| Wisconsin | 447 | 0.14 | 1.40 |
| West Virginia | 11 | 0.10 | 0.79 |

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Climate Adaptation Policy

- Each state has particular characteristics and capacities.

Preparing for Climate Change in Colorado

Change State

This page provides an overview of the steps Colorado is taking to prepare for the impacts of climate change.

On November 5, 2007, Governor Bill Ritter called on the state to prepare for and adapt to climate changes that cannot be avoided and the state released the **Colorado Climate Action Plan: A Strategy to Address Global Warming**. On April 22, 2008, Governor Ritter created the state's Climate Change Advisory Panel and called on the panel to make recommendations on how the state could achieve the goals of Colorado's Climate Action Plan, including by developing adaptation strategies (**Executive Order B007 08**).

[Read More](#)

STATE AGENCY ACTION OVERVIEW

LOCAL SUPPORT OVERVIEW

Colorado



Colorado Climate Plan 2018 Update - State Level Policies and Strategies to Mitigate and Adapt
Finalized: July 2018

Colorado Climate Preparedness Project: Final Report
Finalized: 2011

SUBMIT AN UPDATE ABOUT THIS STATE'S PROGRESS

State Law and Policy

| Resource Name | Resource Type | Date |
|--|--------------------|---------------|
| Colorado Executive Order 2017-015: Supporting Colorado's Clean Energy Transition | Law and Governance | July 11, 2017 |
| Colorado HB 13-1293: Creates climate change position in executive branch | Law and Governance | May 28, 2013 |

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Climate Adaptation Policy

For example, coastal states emphasize the risk of sea level rise.



Preparing for Climate Change in Florida

Change State

This page provides an overview of the steps Florida is taking to prepare for the impacts of climate change.

On July 13, 2007, Governor Charlie Crist established the Florida Governor's Action Team on Energy and Climate Change and directed it to create a comprehensive Energy and Climate Change Action Plan (**Executive Order 07-128**). The executive order directed that the Action Plan include strategies to reduce greenhouse gas emissions and, in a second phase, long-term strategies for reducing climate impacts to society, public health, the economy, and the natural environment. Six work groups were formed to develop recommendations for the Action Plan, including the Adaptation Technical Work Group, which was responsible for developing the adaptation portion of the Action Plan.

[Read More](#)

STATE AGENCY ACTION OVERVIEW 30/26

LOCAL SUPPORT OVERVIEW

Florida



Florida's Energy and Climate Change Action Plan
Finalized: October 15, 2008

SUBMIT AN UPDATE ABOUT THIS STATE'S PROGRESS

State Law and Policy

| Resource Name | Resource Type | Date |
|---|--------------------|---|
| Florida Senate Bill (S.B.) 2514: Resilient Florida Trust Fund | Law and Governance | May 13, 2021 |
| Florida Senate Bill (S.B.) 1954: Statewide Flooding and Sea-Level Rise Resilience | Law and Governance | May 13, 2021 |
| Florida Senate Resolution 1572 | Law and Governance | March 5, 2020 |
| Florida Senate Bill 178: An Act Relating to Public Financing of Construction Projects | Law and Governance | March 11, 2020 (effective July 1, 2021) |

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Climate Adaptation Policy

- Each state has particular characteristics and capacities.

Preparing for Climate Change in California

Climate State

This page provides an overview of the steps California is taking to prepare for the impacts of climate change.

California is engaged in both directly planning for climate impacts and incorporating climate adaptation into its existing policies and programs. On November 14, 2008, Governor Arnold Schwarzenegger called on the California Natural Resources Agency (CNRA) to develop a state Climate Adaptation Strategy in coordination with public and private entities at the local, regional, state and federal levels (**Executive Order S-13-08**). CNRA organized state agencies into seven resource-based sector working groups. Each working group identified both short- and long-term adaptation goals and a process for reporting on progress.

Read More

STATE AGENCY ACTION OVERVIEW

LOCAL SUPPORT OVERVIEW

California

Safeguarding California Plan: 2018 Update
Finalized: January 30, 2018

Safeguarding California: Reducing Climate Risk - 2014 Update to the 2009 CA Climate Adaptation Strategy
Finalized: July 31, 2014

California 2009 Climate Adaptation Strategy
Finalized: 2009

[SUBMIT AN UPDATE ABOUT THIS STATE'S PROGRESS](#)

State Law and Policy

| Resource Name | Resource Type | Date |
|--|--------------------|--------------------|
| California Dedicates \$15 Billion for Climate Action and Protecting Frontline Communities | Law and Governance | September 23, 2021 |
| California AB-65 Coastal protection: climate adaptation: project prioritization: natural infrastructure: local general plans | Law and Governance | September 27, 2019 |
| CA EO N-05-19 CAL FIRE to provide recommendations on wildfire prevention and mitigation, including at-risk populations | Law and Governance | January 8, 2019 |
| California SB 901 - Wildfire Preparedness and Response | Law and Governance | September 21, 2018 |

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Climate Adaptation Policy: Table 6 Panel B

$$\begin{aligned}
 Y_{i,t} = & \alpha + \beta_1 * Skilled\ labor\ mobility_{i,t} + Frequent\ natural\ disasters_{s,t} \\
 & + \beta_2 * Skilled\ labor\ mobility_{i,t} + Frequent\ natural\ disasters_{s,t} + Climate\ Adaptation\ Policy_{s,t} \\
 & + \beta_3 * Climate\ Adaptation\ Policy_{s,t} + \gamma * X_{i,t-1} + \epsilon \quad (6)
 \end{aligned}$$

Panel B: The impact of climate adaptation policy

| | Liquidity buffers | Product market outcomes | | |
|--|-------------------|-------------------------|-----------|-----------|
| | | Sales growth | MSG_SIC | MSG_FF |
| | (1) | (2) | (3) | (4) |
| <i>Climate adaptation policy * Skilled labor mobility * Frequent climate disasters</i> | -0.0017* | 0.0030 | 0.0032 | 0.0040 |
| | (-1.83) | (0.79) | (0.86) | (1.05) |
| <i>Skilled labor mobility * Frequent climate disasters</i> | 0.0102*** | 0.0089*** | 0.0089*** | 0.0092*** |
| | (28.73) | (6.20) | (6.32) | (6.45) |
| <i>Climate adaptation policy</i> | 0.0201*** | -0.0237 | -0.0219 | -0.0236 |
| | (3.11) | (-0.89) | (-0.84) | (-0.89) |
| Controls | Yes | Yes | Yes | Yes |
| Year fixed effects | Yes | Yes | Yes | Yes |
| Industry fixed effects | Yes | Yes | Yes | Yes |
| Observations | 42464 | 40944 | 40944 | 40944 |
| Adjusted R ² | 0.3589 | 0.0532 | 0.0422 | 0.0431 |

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Inevitable Disclosure Doctrine

- The Inevitable Disclosure Doctrine (IDD) restrain the mobility of workers beyond the effects of the noncompete agreement
- Klasa, Ortiz-Molina, Serfing, and Srinivsan (JFE, 2018) and Sanati, JFQA, 2024)

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IDD-Continued

- If the new employment would inevitably lead to the disclosure of the former employer's trade secrets to a competitor, state courts can prevent the employee from working for the competitor (Flammer and Kacperczyk, Strategic Management Journal, 2019)
- **The adoption (rejection) of IDD restricts workers from joining rival companies, decreasing (increasing) labor mobility**

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Inevitable Disclosure Doctrine (IDD)

- If the new employment would inevitably lead to the disclosure of the former employer's trade secrets to a competitor, state courts can prevent the employee from working for the competitor (Flammer and Kacperczyk, Strategic Management Journal, 2019)
- **The adoption (rejection) of IDD restricts workers from joining rival companies, decreasing (increasing) labor mobility**

We focus on **Rejection of the IDD**, which increases labor mobility

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The Impact of IDD

- The rejection of the IDD indicates relaxing of labor mobility constraint.

Table 6: IDD and natural disasters
Panel A reports the year of adoption (rejection) of the inevitable disclosure doctrine (IDD) for U.S. states. The table is sourced from Klass et al. (2018), Flammer and Kacperczyk (2019) and Swann (2024). Panel B reports the regression results of firms' cash holdings on labor mobility and natural disasters. The results using different model specifications are reported. All the variables are winsorized at 1% and 99% to mitigate the outlier problem. Financial firms (SIC 6000-6999) and utilities (SIC 4900-4999) are excluded from analyses. Industry fixed effect are indicators for three-digit SIC code. All the standard errors are clustered by firm. t -statistics are reported in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Panel A: The adoption or Rejection of the Inevitable Disclosure Doctrine

| State | Decision | Year |
|---------------|----------|------|
| Arkansas | Adopt | 1997 |
| | Reject | 2009 |
| California | Reject | 1944 |
| Connecticut | Adopt | 1996 |
| Delaware | Adopt | 1964 |
| Florida | Adopt | 1960 |
| | Reject | 2001 |
| Georgia | Adopt | 1998 |
| | Reject | 2013 |
| Illinois | Reject | 1989 |
| | Adopt | 1995 |
| Indiana | Adopt | 1995 |
| Iowa | Adopt | 1991 |
| Kansas | Adopt | 2006 |
| Louisiana | Reject | 1967 |
| Maryland | Reject | 2004 |
| Massachusetts | Adopt | 1994 |
| | Reject | 2012 |
| Michigan | Adopt | 1966 |
| | Reject | 2002 |
| Minnesota | Adopt | 1986 |
| | Reject | 1992 |

| | | |
|----------------|--------|------|
| Missouri | Adopt | 2000 |
| New Jersey | Reject | 1980 |
| | Adopt | 1987 |
| New York | Adopt | 1919 |
| | Reject | 1999 |
| North Carolina | Reject | 1976 |
| | Adopt | 1996 |
| Ohio | Adopt | 2000 |
| Pennsylvania | Adopt | 1982 |
| Texas | Adopt | 1993 |
| | Reject | 2003 |
| Utah | Adopt | 1998 |
| Virginia | Reject | 1999 |
| Washington | Adopt | 1997 |
| | Reject | 2012 |

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Empirical Results: The Impact of IDD

$$Y_{i,t} = \alpha + \beta_1 * Labor\ mobility\ (IDD\ rejection)_{s,t} + \beta_2 * Labor\ mobility\ (IDD\ rejection)_{s,t} * Frequent\ natural\ disasters_{s,t} + \beta_3 * Frequent\ natural\ disasters_{s,t} + \gamma * X_{i,t-1} + \epsilon$$

Panel B: IDD and Frequent climate disasters

| | Liquidity buffers | | Product market outcomes | |
|--|---------------------|---------------------|-------------------------|---------------------|
| | (1) | (2) | (3) | (4) |
| <i>Labor mobility (IDD rejection) * Frequent climate disasters</i> | 0.0173*** (4.32) | -0.0249* (-1.83) | -0.0200 (-1.50) | -0.0232* (-1.72) |
| <i>Labor mobility (IDD rejection)</i> | 0.0214*** (5.11) | 0.0143 (1.18) | 0.0093 (0.79) | 0.0127 (1.05) |
| <i>Frequent climate disasters</i> | -0.0016 (-0.74) | -0.0081 (-1.47) | -0.0075 (-1.39) | -0.0065 (-1.18) |
| Controls | Yes | Yes | Yes | Yes |
| Year fixed effects | Yes | Yes | Yes | Yes |
| Industry fixed effects | Yes | Yes | Yes | Yes |
| Observations | 122140 | 117702 | 117702 | 117702 |
| Adjusted R ² | 0.3597 | 0.0510 | 0.0422 | 0.0430 |

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Main Findings

- Skilled labor mobility exposed to climate disasters increases liquidity buffers whereas routine-task labor does not
- Skilled labor mobility exposed to frequent disasters depresses sales growth and market shares while routine-task labor does not
- The state-level climate adaptation policies can mitigate the detrimental effects of severe climate disasters

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**Thank You for Your
Attention**

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Keynote Speech

THE WISDOM OF THE CROWD AND HIGHER-ORDER BELIEFS

YI-CHUN CHEN^A, MANUEL MUELLER-FRANK^B, AND MALLESH M. PAI^C

ABSTRACT: We propose a new simple procedure called *Population-Mean-Based Aggregation* (PMBA) that enables a principal to “aggregate” information about an unknown state of the world from agents without understanding the information structure among them. PMBA only requires agents to communicate their beliefs about the state, and some agents to communicate their expectations of the population average belief. In a large population, for any finite number of possible states, and under weak assumptions on the information structure, including individual agents’ beliefs being possibly misspecified, we show that PMBA always infers the true state. We provide evidence that our procedure performs well on the experimental data of [Prelec, Seung, and McCoy \(2017\)](#).

KEYWORDS: aggregating beliefs, higher-order beliefs, wisdom of the crowd.

JEL CLASSIFICATION: D82, D83.

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JANUARY 27, 2024

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1. INTRODUCTION

A long-held belief, starting from at least Aristotle, is that of the “wisdom of the crowd.” This refers to the idea that even though some event may be uncertain, and the information each individual has may be noisy, the “aggregate” of individual beliefs is accurate. Such aggregation obviously has large social and private value, especially when it concerns important social or economic events. As a consequence, there has been substantial effort toward both improving existing institutions (e.g., polling), and designing and implementing new methods (e.g., prediction markets).¹ At a theoretical level, then, it is important to understand the limits of such an exercise: without specific assumptions on the nature of the information agents have, how can we aggregate it, and how much can we learn from it? In short, how wise is the crowd, and how can we tap into its wisdom?

To fix ideas, imagine a population of interest and an event whose realization is uncertain. Individual agents each see a signal that is informative of the realization. Agents also understand the informational environment, i.e., what this signal implies for their own beliefs and the implied distribution of other agents’ beliefs. Even assuming away the difficulties (both logistical and strategic) of eliciting the relevant information from the population, the question we address in this paper is: Does there exist a general procedure to aggregate individuals’ information without knowledge of the information structure among them?

One important insight from the existing literature is that even if the state is binary, and agents’ signals are i.i.d. conditional on the state, knowing the first-order beliefs of even an infinite set of agents is generally insufficient to learn the state.² In particular, even in these idealized conditions, most agents may be wrong, i.e., most agents may place a posterior probability of larger than 50% on the incorrect state of the world. The main takeaway from Arieli, Babichenko, and Smorodinsky (2017) is that without further strong assumptions about the signal structure, the principal cannot learn the state of the world, even with an infinite number of agents in the population. In the terminology of econometrics, there is an “identification problem.”

We answer this question by means of a simple procedure we name *Population-Mean-Based Aggregation* (herein PMBA). Our procedure uses agents’ beliefs about the uncertain event as well as some agents’ expectation of the average of others’ beliefs. Under weak conditions on the information structure generating the agents’ signals, this procedure

¹There is also a large and influential body of literature studies the possibility of such aggregation within our current institutions, e.g., voting (see e.g., Austen-Smith and Banks (1996), Feddersen and Pesendorfer (1997)), markets (e.g., Grossman and Stiglitz (1980)) and social networks (see e.g., Golub and Jackson (2010), Acemoglu, Dahleh, Lobel, and Ozdaglar (2011)).

²See Prelec, Seung, and McCoy (2017) and Arieli, Babichenko, and Smorodinsky (2017).

fully aggregates the information among agents in a large population. In other words, using information from agents, solely about agents’ own beliefs, and their expectation of the average belief in the population (herein referred to as population average belief), but with no other knowledge of the underlying information structure, we can fully learn the underlying state. Importantly, we show that our procedure is *robust*: it allows for aggregation of beliefs even when beliefs are misspecified at both the individual and aggregate levels. That is to say, we start from a “correctly specified” model where agents have beliefs and higher-order beliefs from a common prior and show that our PMBA procedure correctly aggregates agents’ information without knowledge of the underlying common prior. We then show that this procedure continues to correctly aggregate even if individual beliefs are *arbitrarily* misspecified, as long as *in aggregate*, the misspecification is not too large, in a sense that we make precise in what follows. As a result, our procedure is a viable candidate to be used in practice.

The idea of eliciting and using higher-order beliefs is not new: In our context, we note the elegant “Surprisingly Popular” (herein, SP) procedure developed by [Prelec, Seung, and McCoy \(2017\)](#) (henceforth PSM), who pioneered the use of second-order beliefs for the purpose of information aggregation (however their procedure aggregates information on a narrow set of information structures—we discuss this in more detail in Section 3.1). Another close comparison is to the Surprisingly Confident (SC) procedure of [Wilkening, Martinie, and Howe \(2021\)](#), who also leverage the linearity of the correspondence between beliefs and higher-order beliefs, and its implication for the ordering of average beliefs and average second-order beliefs in each state (see also Theorem 1.4 of [Prelec, Seung, and McCoy \(2017\)](#) and contemporaneous approaches by [Libgober \(2021\)](#) and [Prelec and McCoy \(2022\)](#), described in Section 6). A key differentiator is that SP/SC are both only guaranteed to work under binary states, i.e., two possible states of the world, and require stronger assumptions more generally, as we discuss below.

In line with this literature, it is important to note that we are solely using agents’ beliefs and expected higher-order beliefs. Such information has been robustly elicited in laboratory and real-world settings, and therefore, our procedure is amenable to implementation. We show in Section 3 that our procedure performs well on the experimental data of [Prelec, Seung, and McCoy \(2017\)](#). By contrast, theoretically feasible procedures that ask agents for more detailed information such as state-contingent beliefs (for example, the agent’s expectation of the average belief in the population *conditional* on the true state of the world) may not be reliable or even feasible in practice. There is a wealth of evidence that agents struggle to report richer details (such as state-contingent beliefs).³

³For example, in [Li \(2017\)](#), he states: “More generally, laboratory subjects find it difficult to reason state-by-state about hypothetical scenarios ([Charness and Levin, 2009](#); [Esponda and Vespa, 2014](#); [Ngangoué and](#)

Similarly, we may not be able to elicit the underlying prior, i.e., we take seriously the idea that the prior/ ex-ante stage is an “as-if” assumption.⁴

1.1. Summary of Model and Results

Our baseline model, detailed in Section 2, considers general environments with a finite number of states of the world and many agents. We suppose that agents observe signals according to an information structure that is *unknown to the principal* but understood among the agents themselves. Theorems 1 and 2 show that, under general conditions, full aggregation can be obtained using the population’s first-order beliefs and, from (at least) as many agents as there are states, agents’ expectations of the average belief of the population.⁵ In particular, we explain how to more fruitfully use higher-order beliefs when there are multiple states (indeed, PSM themselves point out the difficulties of extending the SP procedure when there are three or more states, and we recap the issues with illustrative examples in Section 3.1).

The basic idea is intuitive and can be described verbally, especially in the case of binary states (Theorem 1, Section 2.2). For simplicity, suppose every agent sees a conditionally i.i.d. signal about the unknown state according to an information structure known to them but unknown to the principal. Now suppose the principal elicits, from each agent, their belief about the state. By the law of large numbers, the population average belief must equal the expected belief in that state.⁶ If the principal knows the expected belief in each state, then they would be done: they could compare the elicited average to the expected belief in each state.⁷ However, they do not know this mapping of the state to the expected beliefs.

Suppose the principal asked two agents with different beliefs for their expectation of the average belief of the population. By the law of iterated expectations, each reports the weighted average of the expected belief conditional on the state, weighted by their (different) likelihood of the two states. This presents the principal a simple system of two linear equations in two unknowns; thus the expected beliefs in each state can be uniquely recovered from the information from these two agents. We show in what follows that this basic idea is much more applicable.

Weizsäcker, 2021). This mental process, often called “contingent reasoning,” has received little formal treatment in economic theory. In subsequent work, Martínez-Marquina, Niederle, and Vespa (2019) investigate axioms governing contingent reasoning in single-agent decision problems.”

⁴See for example the discussion on page 926 in Gul (1998).

⁵Section 3 discusses our rationale for choosing what information we ask agents for. Specifically, the ex-ante prior is purely a modeling device. Agents are informed, i.e., the procedure interacts with agents at the interim stage. Asking agents for their priors is not possible.

⁶This is an important point, see Remarks 3 and 4 below for further discussion.

⁷For informative signals, these expected beliefs must be different in the two states.

In particular, we consider settings where agents understand their own information, but misperceive the distribution of other agents' beliefs etc.— see Theorem 3, Section 2.4. This idea of the misperception of others' information is related to the environments considered in Eyster and Rabin (2010), Bohren (2016), and Frick, Iijima, and Ishii (2020). It is arguably a more reasonable description of the real world. We consider 2 possible states of the world, and allow that individual agents may be arbitrarily misinformed about the state contingent average beliefs of others. We only assume 1) this misspecification about the state contingent average beliefs of others is stochastically independent of their own belief about the unknown state, and 2) *on average*, the agents' misspecification is smaller than the difference of the state contingent average beliefs. We show that in this case a variant of PMBA still successfully aggregates agents' information. Indeed, this is the basis of the procedure that we take to the data in Section 3. Further, as we discuss there, SP is not robust to this sort of misspecification.

While our present paper is theoretical, the idea of using agents' higher-order beliefs has been successfully implemented in practice, both in the lab and in the field. Section 3 discusses more generally the rationale for the information we choose to ask agents for, and possible pitfalls in asking for richer information. Section 3.1 studies the implications on the SP procedure. In particular, we demonstrate how our analysis implies that the procedure is more generally applicable than previously understood, and how our procedure is applicable when it is not.

The original SP paper (Prelec, Seung, and McCoy, 2017) showed via lab experiments that such procedures have promise not just in theory, but also in practice. However, a reader might be concerned that our procedure uses the higher-order belief information in a more exacting way than the SP procedure and related aggregation schemes: in particular the SP procedure for binary states of the world uses only the ordinal part of this information, while we attempt a matrix inversion with these beliefs. With this in mind, we provide evidence that our procedure has real-world plausibility. In Section 3 we revisit their lab-experiments.⁸ We show that our procedure performs comparably with the SP procedure even when the set of states of the world is binary (i.e., where our procedure offers no advantage over the SP procedure).

A majority of this paper considers unincentivized procedures (i.e., we are not considering the incentives of agents to report truthfully, only aggregating their reports assuming that they are). Nevertheless, our procedure naturally extends to a mechanism in which truthful reporting is a Bayes-Nash equilibrium among agents, analogous to the connection between SP and Bayesian Truth Serum—see Section 4.2 for details. In the remainder

⁸We are grateful to the authors, Drazen Prelec, H Sebastian Seung and John McCoy for sharing their original data with us.

of the paper, the use of the word *elicit* only refers to unincentivized elicitation, and we refer to our aggregation scheme as a procedure rather than a mechanism to clarify that there are no incentives. We also show how to adapt our method when only asking for “votes” from most agents (Proposition 5, Section 4.1), e.g., the state they perceive to be most likely (this was also the main exercise considered in Prelec, Seung, and McCoy (2017)).

In Section 5, we attempt to understand the full power and limitations of aggregation with higher-order beliefs. PMBA shows that with an infinite population, access to agents’ first-order and (a few agents’) second-order beliefs can suffice. We consider the case of a finite population (in particular, where signals are not conditional i.i.d.). We show that if the principal can extract the full hierarchy of agents’ higher-order beliefs, they can learn the “full information posterior”, i.e., the posterior of an omniscient agent who shares the agents’ prior belief and sees all their individual signals, despite the principal not knowing the prior and only collecting information about posteriors. Conversely, we show that if the principal only sees a finite prefix of the hierarchy (e.g., only the first k beliefs) of a population, then there remains an “identification problem,” i.e., there exist two different information structures that could both result in this exact realization. In short, higher-order beliefs cannot achieve full Bayesian aggregation in the case of a finite population unless we can ask agents for their entire hierarchy, which we consider unrealistic/ impractical. Section 6 discusses the related literature in further detail and concludes.

2. POPULATION-MEAN-BASED AGGREGATION

In this section, we introduce our core model; we describe our Population-Mean-Based Aggregation procedure (herein PMBA) and prove its validity.

2.1. General Model and Notation

There is an infinite population of agents. To this end, let $N = \{1, 2, \dots\}$ be a countably infinite set of agents, with i denoting a generic agent. There are a finite set of L states of the world $\Omega = \{\omega_1, \omega_2 \dots \omega_L\}$. The space of feasible beliefs over Ω , denoted as $\Delta(\Omega)$, is the L dimensional simplex Δ^L .

Each agent i observes a signal in a set S with the associated sigma field \mathcal{F} . To allow for correlation, let us denote the common prior over $\Omega \times S^N$ by P . Given that an agent observes some signal $s \in S$, their posterior belief that the state is ω is given by $P(\omega|s)$. Let us denote the posterior beliefs of agent i by $\mu_i(s) \in \Delta^L$ and $\tilde{\mu}_i$ as the associated random variable. In what follows, the actual signal seen by the agent will often be irrelevant since we are concerned with their beliefs. We will, therefore, suppress the dependence on the signal in our notation. We define

$$\bar{\mu}_i(\omega) = \mathbb{E}[\tilde{\mu}_i|\omega].$$

In words, $\bar{\mu}_i(\omega)$ is the expected belief of agent i in state ω .

Finally, we define $\bar{\mu}(\omega) = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n \bar{\mu}_i(\omega)$.⁹ This is the expected population average belief in state ω .

Given a vector $x \in \Delta(\Omega)$, we will let x_ω refer to the component corresponding to ω ; i.e., $\mu_{i,\omega}$ is the belief of agent i that the state is ω , $\tilde{\mu}_{i,\omega}$ is the random variable of agent i 's belief that the state is ω , etc.

2.2. Binary States

In this section, we consider the case of binary states, i.e., $L = 2$. This procedure will form the core of our general analysis.

We let $P_{i,\omega}^S$ denote the induced marginal distribution of P over signals for agent i in state ω . We also abuse notation so that μ_i represents agent i 's belief that the state is ω_1 : since there are only two states, their belief that the state is ω_2 is $1 - \mu_i$.

ASSUMPTION 1 (No Aggregate Uncertainty). *We make the following assumptions on P :*

- (1) *Imperfectly informed agents: for each agent i , we have that $P_{i,\omega}^S$ and $P_{i,\omega'}^S$ are mutually absolutely continuous with respect to each other.*
- (2) *Limited correlation: For every $\varepsilon > 0$, there exists a finite $n(\varepsilon)$ such that conditional on the state, for any agent i , all but at most $n(\varepsilon)$ agents' signals are ε -independent of agent i 's signal. Formally:*

$$\forall \varepsilon > 0, \exists n(\varepsilon) \in \mathbb{N} \text{ s.t. } \forall i, \exists N_i \subseteq N, |N_i| \leq n(\varepsilon),$$

$$\forall j \in N \setminus N_i, \forall E, E' \in \mathcal{F}, \forall \omega : |P(\tilde{s}_{i,\omega} \in E, \tilde{s}_{j,\omega} \in E' | \omega) - P_{i,\omega}^S(E)P_{j,\omega}^S(E')| \leq \varepsilon.$$

This assumption may seem a little dense so some discussion is useful. The first condition is simply to make the problem interesting/non-trivial: after all, if an agent is perfectly informed then aggregation is trivial. Next, we need an assumption regarding the correlations between agents' signals. Intuitively, assuming conditionally i.i.d. signals is likely overly strong and does not capture the idea that agents' information may be correlated. However, we cannot allow for arbitrary correlation—for example, if all agents' information is perfectly correlated, then effectively, there is only one unique opinion in the population, and no further aggregation is possible. Our notion allows for “enough independence,” thereby roughly guaranteeing that for any agent, most other agents' signals are approximately independent conditional on the state. This ensures that the law of large numbers holds, so that the population average belief is a deterministic function the

⁹Note that without further assumptions, it is possible that this limit might not exist and/or might be different depending on the ordering of agents. We ignore these possibilities. Simple sufficient conditions (e.g., conditional i.i.d. signals) rule out these concerns with an infinite population, and they are moot for any arbitrarily large finite population.

of state, i.e., in the notation of our model, in any state ω we have that

$$\text{as } n \rightarrow \infty, \quad \frac{1}{n} \sum_{i=1}^n \mu_i \rightarrow_P \bar{\mu}(\omega).$$

We emphasize that while this assumption relaxes the conditional i.i.d. assumption that is regularly made, it is a non-trivial assumption. Whether it is sensible or not depends on the context. For instance, in a political context, if all agents are drawing their information from the same few noisy news sources, then there will be correlations in the beliefs and our assumption would not be well-motivated for such a setting. Conversely, if we are considering subjective but idiosyncratic product reviews, then the assumption is far more natural.

As we will see in what follows, our PMBA procedure requires that the population expected beliefs differ in the two states, i.e., $\bar{\mu}(\omega_1) \neq \bar{\mu}(\omega_2)$. This is a very weak condition, but one that one may nevertheless worry about. In principle, different distributions of beliefs may nevertheless have the same expected belief. The following assumption delivers this: it requires that all agents individually are at least minimally informed. Formally, this is defined with respect to the total variation metric. This is akin to a normalization—it simply ensures that we do not have infinitely many arbitrarily uninformed agents.

To this end, we need some more notation: let $G_{i,\omega}$ denote the induced distribution of private beliefs of agent i in state ω . We will abuse notation and also let it denote the cumulative distribution function of the belief of agent i in state ω .

ASSUMPTION 2 (Minimal Information). *We assume that there exists $\delta > 0$, such that for any agent i , the total variation distance between G_{i,ω_1} and G_{i,ω_2} is uniformly bounded below by δ , i.e.,*

$$\exists \delta > 0 \text{ s.t. } \forall i : \sup_{E \subseteq [0,1]} |G_{i,\omega_1}(E) - G_{i,\omega_2}(E)| \geq \delta.$$

The first main result of our paper shows the power of the PMBA procedure as described in Procedure 1.

THEOREM 1. *Suppose the information structure among agents satisfies Assumptions 1 and 2. Then the PMBA procedure (Procedure 1) correctly recovers the true state of the world almost surely.*

Before we discuss a sketch of the proof and some directions to generalize the procedure, some remarks about the procedure and the result are in order.

REMARK 1. *Assumptions 1 and 2 jointly are sufficient for PMBA to be valid, but it can be shown that they are not necessary. A full characterization is easy to describe but perhaps unsatisfying—as long as the information structure is such that the law of large numbers holds, i.e., in any*

PROCEDURE 1. Population-Mean-Based Aggregation, 2 states

(1) Elicit, from each agent $i \in N$, their belief μ_i . Calculate $\hat{\mu}$ defined as:

$$\hat{\mu} = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n \mu_i$$

(2) Select two agents, A and B, such that $\mu_A \neq \mu_B$. Elicit from each of these agents their expectation of the average posterior beliefs in the population, i.e., denoting s_i as the signal seen by agent i , elicit for each $i = A, B$:

$$\alpha_i = \lim_{n \rightarrow \infty} \mathbb{E} \left[\frac{1}{n} \sum_{j=1}^n \tilde{\mu}_j \mid s_i \right].$$

(3) Calculate $\bar{\mu}(\omega_1), \bar{\mu}(\omega_2)$ as:

$$\begin{pmatrix} \bar{\mu}(\omega_1) \\ \bar{\mu}(\omega_2) \end{pmatrix} = \begin{pmatrix} \mu_A & 1 - \mu_A \\ \mu_B & 1 - \mu_B \end{pmatrix}^{-1} \begin{pmatrix} \alpha_A \\ \alpha_B \end{pmatrix} \quad (1)$$

Note that the right-hand side is elicited from the agents directly; and since $\mu_A \neq \mu_B$, the middle matrix is invertible.

(4) Recover the state by comparing $\hat{\mu}$ calculated in step 1 with $\bar{\mu}(\cdot)$ calculated above. The state of the world is ω s.t. $\hat{\mu} = \bar{\mu}(\omega)$.

state ω , $\text{plim}_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n \tilde{\mu}_i = \bar{\mu}(\omega)$ (which is an implication of Assumption 1); and $\bar{\mu} \neq \bar{\mu}'$ (respectively, an implication of Assumption 2), it will be clear from the proofs that PMBA will successfully recover the true state of the world. Expressing this (in particular, concentration) in terms of primitives, however, appears out of reach.

REMARK 2. It is worth re-emphasizing that the PMBA procedure does not require any knowledge of the underlying information structure among agents—a different way of interpreting our result, therefore, is that as long as these conditions are satisfied, knowledge of the information structure among agents is not needed to fully aggregate their information. That said, it is possible to construct examples where these conditions are violated but a principal who knows the information structure among agents can aggregate their information. For example, one can construct settings, e.g., with two states, such that $\bar{\mu}(\omega_1) = \bar{\mu}(\omega_2)$, but the population distribution of posteriors in these two states are different. A Bayesian principal who knows this would be able to perfectly aggregate agents' information, but PMBA would clearly fail in such a setting.¹⁰

REMARK 3. It is important, both conceptually and technically, to note that our assumptions (in particular, Assumption 1 part (2)) imply that $\bar{\mu}(\omega)$ is a degenerate random variable, i.e., a

¹⁰As a simple illustration, consider the following example: there are two states, $\{\omega_1, \omega_2\}$, and two signals, $\{s_1, s_2\}$. All agents' prior is that either state is equally likely. In state ω_j , agent i receives the "correct" signal s_j with probability $\frac{1}{2} + e^{-i}$, and with the remaining probability receives the incorrect signal s_{3-j} (this clearly violates Assumption 2). PMBA will fail in this setting: the expected population average belief in both states is $\frac{1}{2}$. However, by observation, a Bayesian who knows the signal structure can learn from the agents' beliefs.

deterministic function of the state. Note that our PMBA procedure would fail otherwise because it recovers $\mathbb{E}[\bar{\mu}(\omega)|\omega]$, which may not equal the realization if $\bar{\mu}(\omega)$ is a non-degenerate random variable. One can construct settings where PMBA would fail, but a Bayesian principal who knew the information structure among agents would learn the true state of the world (for example, if $\bar{\mu}(\omega)$ is a non-degenerate r.v. but has disjoint support across the states).

Conceptually, our assumption puts additional structure on what we mean by a “state of the world.” For example, consider a setting where agents’ signals are exchangeable conditional on the state. In this case, DiFinetti’s theorem tells us that the signals are i.i.d. conditional on state and an additional latent random variable. Our assumption then rules out the existence of additional latent random variables on which $\bar{\mu}(\omega)$ depends—the description of the state must be rich enough so that $\bar{\mu}(\cdot)$ is a deterministic function of ω .

A formal proof is given in Appendix A. Here, we want to discuss how the various assumptions play a role in the result. As we pointed out earlier, Part (1) of Assumption 1 is mainly a normalization. The key processing step in this procedure is Step 3. By the law of iterated expectations, an agent i with beliefs μ_i will report, in step (2),

$$\alpha_i = \mu_i \bar{\mu}(\omega_1) + (1 - \mu_i) \bar{\mu}(\omega_2).$$

Our procedure inverts this relationship to learn the unknown mapping from ω to $\bar{\mu}(\omega)$, using (1).

Step (4) then recovers the state by comparing the calculated $\bar{\mu}(\cdot)$ to the elicited $\hat{\mu}$. Part (2) of Assumption 1 ensures that an appropriate law of large numbers holds so that in state ω , $\hat{\mu} = \bar{\mu}(\omega)$, almost surely. Assumption 2 is used to guarantee that $\bar{\mu}(\omega_1) \neq \bar{\mu}(\omega_2)$. Therefore, the elicited $\hat{\mu}$ will almost surely match $\bar{\mu}(\omega)$ for a unique state, and the procedure thus successfully concludes. Taken together, therefore, these give us our result.

To be clear, the model and result of this section are explicitly written for a common prior model where the agents perfectly understand the signal structure among themselves, and only the principal does not. This knowledge of each other’s information among agents may seem daunting or even implausible in applied contexts.

To that end, our procedure does not require agents to have an exact understanding of the distribution of signals among fellow agents. We can observe that it is sufficient for each agent to understand the aggregate implications perfectly; that is to say, the expected population beliefs in each state, $\bar{\mu}(\cdot)$ since that is all they need to know (other than their own beliefs) to calculate the information that we elicit. This is a substantially lower informational requirement. Indeed, in Section 4.1, we show how even less demanding information may be elicited (e.g., asking most agents what state they think is more likely).

However, even this may seem high—perhaps agents don’t perfectly understand even the aggregate implications of the model—for example, each agent has noisy information

about the true $\bar{\mu}(\cdot)$, rather than exact knowledge of it. In Section 2.4, we show how modifying our procedure allows for population aggregation in such a setting.

2.3. $L > 2$ States

We now consider the case of $L > 2$ states. Intuitively, we generalize the “matrix inversion” ideas of the previous subsection to propose a PMBA procedure that works for L states. However, having more than 2 states introduces some difficulties; notably, the assumptions that guarantee this process works become slightly more demanding. Assumption 1 still applies as written; observe that nothing in that assumption was specific to the case of binary states.

However, a direct generalization of Assumption 2 to more than two states is generally not sufficient to assure that the state-dependent averages are unequal across states. So, we assume the L state generalization of its implication directly.

ASSUMPTION 3. *We assume that P is such that for any two states ω and ω' , $\bar{\mu}(\omega) \neq \bar{\mu}(\omega')$.*

Note that while Assumption 3 is not directly stated in terms of the primitives of our model (P), it should be clear that this is a property that can be easily satisfied.¹¹

Finally, in the 2–state case, we are guaranteed the a.s. existence of two agents who have different beliefs. The appropriate generalization turns out to be the existence of L agents whose beliefs constitute a full rank $L \times L$ matrix so that the analog/generalization of (1) in step (3) can be carried out. The following assumption guarantees this:

ASSUMPTION 4. *We assume that P is such that for any agent i , the support of beliefs $\text{supp}(\bar{\mu}_i)$ contains L distinct points, such that those L beliefs, viewed as L -dimensional vectors, constitute a set of full rank. Alternately and equivalently, we require that the convex-hull of the support has an interior relative to Δ^L .*

One might wonder why such a condition was not required in the case of binary states. The answer is that when $L = 2$, this condition simply reduces to requiring that the support of the agents’ beliefs has at least two points in any state. Part (1) of Assumption 1, however, implies that the support of the agents’ beliefs is the same across states. If this support was degenerate at a single point, we would clearly violate Assumption 2. Note that with $L > 2$, there clearly exist settings that violate Assumption 4 despite satisfying Assumptions 1 and 2; for instance, consider a setting where $L = 3$ so that the simplex is a triangle, but there are only two possible signals, so that there only two induced beliefs.

¹¹For example, in the case of finite conditionally i.i.d signals, this property is generically satisfied in the belief space, in the sense of full measure.

We should note that this condition is weak in at least two ways. First, in the standard genericity sense: a generically chosen set of feasible posteriors will be full rank. Second, and perhaps more interestingly, even in settings where there are fewer signals than states (e.g., agents can run only a unique binary-outcome experiment whose outcome determines their posterior), as long as the agents can acquire enough independent signals (e.g., conduct a sufficient number of independent experiments), the set of feasible posteriors is full rank. We formally state this claim below—the proof essentially repurposes a lemma of Fu, Haghpanah, Hartline, and Kleinberg (2021).

CLAIM 1. *Suppose that signals are conditionally i.i.d. draws from a finite set S , and Assumption 3 is satisfied. If each agent gets $L - 1$ independent draws from the distribution P_ω^S , then Assumption 4 is satisfied with the signal space S^{L-1} .*

We are now in a position to describe our main procedure, Population-Mean-Based Aggregation (PMBA) (Procedure 2 below) and state our main result for this setting.

PROCEDURE 2. Population-Mean-Based Aggregation, L states

- (1) Elicit, from each agent $i \in N$, their belief μ_i . Calculate $\hat{\mu}$ defined as:

$$\hat{\mu} = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n \mu_i$$

- (2) Select L agents $I = \{i_1, i_2, \dots, i_L\}$ such that the $L \times L$ matrix $\mu = (\mu_{i_1}^T, \mu_{i_2}^T, \dots, \mu_{i_L}^T)^T$ is full rank.
 (3) Let s_i be the signal seen by agent i . Elicit from each of these agents their expectation of the average posterior beliefs in the population, i.e., elicit for each $i \in I$:

$$\alpha_i = \lim_{n \rightarrow \infty} \mathbb{E} \left[\frac{1}{n} \sum_{j=1}^n \tilde{\mu}_j \mid s_i \right].$$

Denote $\alpha = (\alpha_{i_1}^T, \alpha_{i_2}^T, \dots, \alpha_{i_L}^T)^T$.

- (4) Solve

$$\bar{\mu} = \mu^{-1} \alpha \tag{2}$$

Note that the right-hand side is elicited from agents directly and therefore $\bar{\mu}$ can be recovered. The state ω is the row of the recovered $\bar{\mu}$ that equals the calculated $\hat{\mu}$.

THEOREM 2. *Suppose $L > 2$ and the common prior P satisfies Assumptions 1, 3, and 4. Then the PMBA procedure (Procedure 2) recovers the true unknown state of nature ω almost surely.*

2.4. Aggregation in Misspecified Information Settings

In practice, agents may have limited and individually incorrect knowledge about the aggregate information structure: in particular, a standard concern is that while agents may have meaningful information that shapes their *personal* beliefs, they nevertheless may be less informed about the belief of others, or even summary statistics of the beliefs of others like the average belief as elicited by our procedure.

Secondly, in practice there will be only a finite number of agents, and therefore it is unlikely that one of the probabilities recovered from the matrix-inversion will exactly match the population average beliefs.

Nevertheless similar ideas can be used to aggregate population information robustly in such settings. In the interests of brevity, we consider the case of two states; the extension to L states is analogous to the previous section.

We maintain the assumptions on the true signal generating process P as in Section 2.2, i.e., Assumptions 1 and 2. Instead of assuming common knowledge of P , we assume that the agents' information about P is misspecified—agent i believes that the state-dependent population averages are $(\alpha_i^{\omega_1}, \alpha_i^{\omega_2}) \in [0, 1]^2$. Note that each agent understands how to interpret their own signal correctly, i.e., the misspecification is purely about knowledge of the population averages. We make the following assumption on the relation between agents' beliefs and the true state-dependent population means.¹²

ASSUMPTION 5. For each agent i and state $\omega \in \Omega$ we have that

$$\alpha_i^\omega = \bar{\mu}(\omega) + \zeta_i^\omega$$

where ζ_i^ω is independent of agent j 's signal s_j conditional on the state, for all agents j . Further we assume that

- Conditional on ω the ζ_i^ω 's satisfy limited correlation across agents.
- For each $\omega \in \Omega$, the ζ_i^ω 's have a common mean, $E[\zeta_i^\omega] = E[\zeta^\omega]$ for all i .
- The distributions of α_i^1 and α_i^2 have disjoint support for all i .

Let us briefly discuss Assumption 5. One implication of this is that each agent understands that conditional on the state their expectation of the population mean is independent of their first-order belief. The assumption that supports are disjoint is necessary,

¹²Even this weaker assumption may be violated in the real world where higher-order belief data is often noisy and boundary reports are common. That said, in practice, it may be more robust to use a linear regression (or a tobit regression to deal with boundary issues) on the lines suggested in Palley and Satopää (2020) and Peker and Wilkening (2023). We do not pursue this line in this paper as we feel it is out of scope, and further even the current mechanism shows promise on real world data as we show in our exercise in Section 3. Nevertheless, this we believe this presents a tantalizing possibility for future work.

keeping in mind that agents are fully Bayesian in their inferences, and they have misspecified knowledge. To be precise, this assumption avoids that their expectation for the population average in state ω_1 is larger than in state ω_2 which would contradict Bayes rationality. The limited correlation is necessary for a law of large numbers to hold.

Importantly note that we do not assume that $\mathbb{E}[\zeta^\omega] = 0$: agents' beliefs about the population averages may be biased even on average (as has indeed been observed in practice), and these biases may be different in different states.¹³

PROCEDURE 3. Misspecified Information Population-Mean-Based Aggregation

- (1) Elicit, from each agent i , their belief μ_i . Calculate $\hat{\mu}$ defined as:

$$\hat{\mu} = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n \mu_i$$

- (2) Elicit from all agents their interim expectation of the average posterior belief over all agents—denote this as α_i for agent i .

Partition agents into two infinite groups A and B such that $\mu_A \neq \mu_B$ (μ_A, μ_B are defined below). Calculate the average of the α_i 's in each group, A, B . For example, without loss, rename agents so that $A = \{1, 2, 3, \dots\}$. Then define

$$\alpha_A = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i \leq n} \alpha_i,$$

$$\mu_A = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i \leq n} \mu_i.$$

and define μ_B, α_B analogously.

- (3) Calculate $\bar{\mu}(\omega)$ as

$$\begin{pmatrix} \bar{\mu}(\omega_1) \\ \bar{\mu}(\omega_2) \end{pmatrix} = \begin{pmatrix} \mu_A & 1 - \mu_A \\ \mu_B & 1 - \mu_B \end{pmatrix}^{-1} \begin{pmatrix} \alpha_A \\ \alpha_B \end{pmatrix} \quad (3)$$

Recover the state as:

$$\omega^0 = \arg \min_{\omega \in \Omega} |\hat{\mu} - \bar{\mu}(\omega)|. \quad (4)$$

We can now state our aggregation result in the misspecified information setting.

THEOREM 3. *Suppose the true signal distribution process satisfies Assumptions 1 and 2, and that the agents' knowledge regarding the information structure satisfies Assumption 5. Suppose further that*

$$\left| E[\zeta^1] \right| + \left| E[\zeta^2] \right| < \bar{\mu}(\omega_1) - \bar{\mu}(\omega_2). \quad (5)$$

¹³See also Remark 5 which clarifies how SC/ SP may fail in this setting.

Then, Misspecified Information Population-Mean-Based Aggregation (Procedure 3) correctly recovers the true state of the world almost surely.

Theorem 3 establishes that indeed the conceptual idea of second-order elicitation successfully overcomes the limited understanding of agents' regarding the private belief distributions of their peers. Compared to the PMBA aggregation under complete knowledge of the information structure, the two infinite groups A and B play the roles of the two agents in the PMBA.

Thus moving to a more realistic assumption of an incomplete and individually incorrect understanding of the information structure necessitates the elicitation of the expected population averages from large groups: we are relying on the crowd for both first- and second-order statistics. The main issue is that individual agents' understanding of the population averages is incorrect. However, by the law of large numbers we can appropriately average these individual agents' reports over two groups, A and B . The average first-order belief, and average report of population average, of agents in each group, will serve as the analogs of the two individuals in the baseline PMBA procedure (Procedure 1).

The basic intuition is that even though individual agents are misinformed about the distribution of their fellow agents' beliefs, on average these errors are not "large" (even though individual agents' errors can well be) relative to the true difference between the two population averages (recall condition (5) in Assumption 5 makes this possible). This ensures that the "closer" recovered $\bar{\mu}(\omega)$ to the realized average population distribution $\hat{\mu}$ is indeed the one corresponding to the state.

Finally, since we are using the "closest" distribution rather than an exact match, this procedure always selects a state (for the earlier procedures, with high probability under a finite population, the procedure would simply fail). It should be clear that under sufficient concentration, a finite population, high probability version of Theorem 3 holds. The exact bounds/guarantees would of course depend on assuming more details of the information structure among agents (e.g. how much correlation), so we do not pursue this avenue further in this paper. Instead, in what follows in Section 3 we report the results of taking this mechanism to the data.

3. PMBA-STYLE PROCEDURES COMPARED TO SP

We now compare our PMBA procedure to the current gold standard, i.e., the Surprisingly Popular (SP) procedure of [Prelec, Seung, and McCoy \(2017\)](#). First we describe SP formally, and explain formally when our procedure can potentially succeed while SP would not. Having described the theoretical properties of PMBA procedures, the natural question for a reader is while this works in theory, could it work in practice. To answer

this in the affirmative, we revisit the experiment of [Prelec, Seung, and McCoy \(2017\)](#). We were graciously provided the raw data for their experiment by the authors to benchmark our own procedure.

3.1. *The Surprisingly Popular procedure*

The original SP procedure considers the case analogous to what we consider in Section 4.1—eliciting from each agent a “guess” or “vote” of the state they consider most likely given their beliefs. It also considers the case of directly eliciting beliefs (see Section 1.4 of the Supplementary Information Section of [Prelec, Seung, and McCoy \(2017\)](#)). More precisely, Section 1.4 of PSM departs from the idea of SP to identify the actual population signal distribution by eliciting the agents’ posterior beliefs and their prediction of other agents’ signal. Our discussion of SP here also applies to the alternative procedure proposed by [Wilkening, Martinie, and Howe \(2021\)](#), Surprisingly Confident (SC).¹⁴ Like SP, SC aggregates when there are only two states; moreover, like PMBA, SC is also based on the linearity of the mapping from first-order belief to its corresponding second-order belief.

We formally define the procedure in our general environment, generalize the existing results, and highlight its limitation relative to PMBA. To start, consider the case of binary states and let the common prior P among agents satisfy Assumption 1 and Assumption 2. In our notation, the SC procedure can be described as follows:

PROCEDURE 4. Surprisingly Popular (SP), 2 states

- (1) Elicit from each agent $i \in N$, their belief μ_i . Calculate $\hat{\mu}$ defined as:

$$\hat{\mu} = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n \mu_i$$

- (2) Select one agent, i , and elicit their expectation of the average posterior beliefs of the population, i.e., denoting s_i as the signal seen by agent i :

$$\alpha_i = \lim_{n \rightarrow \infty} \mathbb{E} \left[\frac{1}{n} \sum_{j=1}^n \tilde{\mu}_j \mid s_i \right].$$

- (3) Compare the realized population average belief $\hat{\mu}$ with agent i ’s expectation α_i . If $\hat{\mu} > \alpha_i$, the state of the world is ω_1 , otherwise it is ω_2 .

This procedure is arguably even simpler than our PBMA procedure. Only one agent is required to report their expectation of the population average and no calculation is

¹⁴SC was named as such because PSM in their original paper referred to the subjects’ posterior beliefs as their confidence. PSM’s main object of analysis uses agents’ guesses or votes, and their expectation of votes of others, while SC directly uses agents’ posteriors and expected population-average posteriors.

required. Nevertheless, SP can recover the truth in the binary state setting as the following result shows.

PROPOSITION 1. *Consider a binary state space. Suppose that the information structure among agents satisfies Assumptions 1 and 2. Then the SP procedure (Procedure 4) correctly recovers the true state of the world.*

Note that this result provides a stronger theoretical guarantee of the applicability of the SP procedure: the original PSM paper considers only a finite set of possible signals. We omit a formal proof. At a high-level, this proceeds straightforwardly from Lemma 2: In a two-state setting (with two or more signals), for any agent i , the induced distribution of beliefs on ω at state ω first-order stochastically dominates the induced distribution of the belief on ω at state $\omega' \neq \omega$. Therefore the population average belief of state ω in ω must be higher than the population average belief of state ω in state ω' . Any agent's expected population average belief must be submitting a convex combination of these two. Therefore if the true state is indeed ω , it will be "surprisingly popular" relative to this expectation of the average beliefs in the population.

In summary, the SP procedure aggregates beliefs in the binary state setting. However, it is not applicable beyond two states. Indeed the original SP procedure cannot be applied to more than binary states without strong additional assumptions. The reason is that, in general, more than two states can be "surprisingly popular", leaving an indeterminacy in how to define the procedure. Even when defining the SP procedure as e.g., picking the state that surpassed by most its expectation, the procedure does not achieve aggregation. PSM impose an additional assumption (see e.g., their Theorem 3), one of "diagonal dominance," roughly:¹⁵ the posterior of an agent who thinks state ω is most likely must also place a higher posterior on ω than any voter voting for $\omega' \neq \omega$ places on ω . We exhibit in Appendix C a counter-example showing that such an assumption is necessary.

At this point, a reader may wonder about whether other iterative or sequential procedures that work for 2 states can successfully aggregate information in settings with more than two states. The short answer is no, for similar reasons as the binary state PMBA cannot be used sequentially—recall Remark 3.

REMARK 4. *One possibility a reader may consider feasible is, in a setting with $L > 2$ states $\{\omega_1, \dots, \omega_L\}$, to run SP sequentially. That is to say first elicit beliefs corresponding to states ω_1 and its complement (states $\{\omega_2, \dots, \omega_L\}$), and run SC/SP on this "binary state" environment. However, note that in this case, the realized population average belief conditional on the second*

¹⁵To be clear, the setting there is one of prediction normalized votes, we are translating to the analogous assumption in our setting for convenience.

“state” ($\{\omega_2, \dots, \omega_L\}$) is not deterministic as a function of this “state” (i.e., it depends on which of the states ω_1 thru ω_L actually realized). As a result the existing procedures do not apply.

Not to belabor the point, but this has two takeaways. Firstly, consider what is often modeled as a binary state space setting: e.g. we wish to aggregate information on the true/false question “Is Chicago the capital of Illinois”. The true underlying state that determines the distribution of beliefs etc may nevertheless be multivalent (e.g. “Chicago is the capital”, “Springfield is the capital”, “Evanston is the capital” etc.). If that is indeed the case, existing approaches may fail even if the question of interest (“Is Chicago the capital of Illinois”) takes only a binary value. PMBA can in principle be applied to such settings, with the proviso that the set of possible states needs to be correctly identified, and corresponding beliefs elicited.

Finally, one might ask whether the existing procedures are robust to misspecification, i.e., when agents’ beliefs about the population average are incorrect (recall Section 2.4). When agents’ information about the population average is misspecified, SP as written can straightforwardly fail (if the agent who is asked for their expected population average belief is misspecified, then the wrong state may be surprisingly popular). We follow the path taken by PSM in their implementation, and consider “average SP”, i.e., ask all agents for the expected population average, average this, and then given the true population average check which state is “surprisingly popular” relative to that. We show by example that there exist settings where the conditions of Theorem 3 are satisfied, so that misspecified Information PMBA can aggregate information, but SP does not.

REMARK 5. *Suppose we have that Assumption 5 is satisfied and further that $\bar{\mu}(\omega_1) = \frac{3}{4}$ and $\bar{\mu}(\omega_2) = \frac{1}{4}$, i.e. the average beliefs in the population are $\frac{3}{4}$ in state ω_1 and $\frac{1}{4}$ in state ω_2 . Further suppose that $E[\xi_2] = 0$ but $E[\xi_1] = \frac{3}{16}$. In other words, on average, agents’ information about the population average in state 2 is correct, but on average, agents believe that in state ω_1 , the average belief in the population is $\frac{15}{16}$.*

By a simple calculation, in state ω_1 , the average expected population belief is $\frac{49}{64}$. This is larger than the population average belief of $\frac{3}{4}$ in ω_1 , so state ω_2 is surprisingly popular in this state, and therefore SP does not correctly aggregate information in this state.

Further, it is easy to verify that the condition of Theorem 3 holds in this case, and therefore, our misspecified Knowledge PMBA procedure correctly aggregates information.

Note that the converse is also possible, i.e. there exist settings where agents have misspecified information in which SP correctly averages state, but which violate our Assumption 5. SP correctly aggregates state whenever the average of agents’ expectations of population average belief lies in the interval of population average beliefs.

3.2. Comparing the Procedures in the Data

To recall, the authors in [Prelec, Seung, and McCoy \(2017\)](#) report the results of using SP to aggregate information in three experiments, namely:

- (1) *Study 1: State Capitals* Respondents got prompts of the form “X is the capital of State y”. Agents had to respond with T/F, and their predicted population average.
- (2) *Study 2: General Knowledge* Respondents were given general knowledge statements, e.g., “Japan has the highest life expectancy in the world.” Responded with T/F, and their predicted population average.
- (3) *Study 3: Dermatology* A collection of dermatologists were recruited and shown pictures of lesions. They were asked to evaluate whether it was benign or malignant (1–6 scale) and what they thought other dermatologists would say (1–11 scale).

Note that in each case the experimenter knows the “ground truth” (in the first two these are easily ascertained, in the latter study the pictures were chosen such that the true malignancy of the tumor was known). Therefore the experimenter can compare whether the aggregation resulting from the procedure results matches the ground truth. This can also be compared with simpler procedures such as population average or majority.

Table 1 reports the results of PMBA on the raw data from these studies, and how these results compare to SP. Formally, we used the procedure described in Section 2.4, i.e. Procedure 3, since that is best suited for “real world” use. We emphasize that our procedure, summarized in Table 1, performs roughly as well as SP.

Note that this remarkable performance of PMBA is despite the studies being in binary settings where our procedure has no theoretical advantage, but as we described earlier may be at a practical disadvantage (as we said earlier, PMBA-style procedures use the cardinal nature of the beliefs since they attempt a matrix-inversion step). Further, [Prelec, Seung, and McCoy \(2017\)](#) does not elicit agents’ beliefs and uses their votes as we described above. However, they do ask agents to report their confidence in their assessment. We interpret this self-reported confidence as agents’ first-order beliefs to perform the matrix inversion step. This achieves good results as shown in Table 1.¹⁶

Formally, for each experiment we conduct the following procedure, which is an amalgam of Procedures 3 and 5

- (1) As we described earlier, we use the reported confidence of each agent as their belief.
- (2) To determine the groups we average over (recall Procedure 3), we split the population into two groups, according to their vote on the question.

¹⁶Documented code has been submitted with the submission.

| Study | Majority | SP | PMBA |
|---------------------|----------|--------|--------|
| Capitals | 62% | 90% | 88% |
| Trivia | 56.2% | 62.5% | 60.29% |
| Dermatology Lesions | 67.5% | 73.75% | 65.15% |

TABLE 1. Table summarizing the results of PMBA performance on Experimental data from [Prelec, Seung, and McCoy \(2017\)](#). Study column denotes which study is under consideration. Majority column reports the fraction of questions in that study where the majority matched the ground truth, analogously for SP and PMBA.

- (3) For each group, we use the average confidence of members as the corresponding “belief”, and the average of the vote shares predicted by members of the group as the corresponding predicted vote share for that group.
- (4) We then solve for the predicted state-contingent expected vote shares given these according to PMBA (recall (3)).
- (5) The predicted state is the one whose vote share is closer to this predicted vote share (3).

We should note two particular difficulties with our implementation: firstly, agents’ confidence in this experiment is a self-report, it is not elicited in an incentivized fashion. This causes no difficulties for [Prelec, Seung, and McCoy \(2017\)](#) in their analysis since this is not an input to SP (recall they are using agents’ “votes” as we described above), they only use it to compare SP against a confidence-weighted majority benchmark. However, for our procedure, we require (some) agents’ first-order beliefs for the matrix-inversion step. Even granting this, a particular difficulty is on the third experiment: it is particularly problematic to interpret confidence as belief since elicitation was on 1 – 6 and 1 – 11 scales. It is an exciting prospect to design and test PMBA directly (with elicited beliefs and population average beliefs) in settings with more than two states. We defer that to future work.

4. EXTENSIONS

In this section, we show how to extend the basic PMBA to other settings. In Section 4.1, we recognize that, in practice, eliciting beliefs from agents is difficult—we show that a variant of the procedure can aggregate information if we only elicit “simple” information (e.g., report which state is thought more likely) from most agents, and elicit more fine-grained information from only a small number of agents. Both extensions are displayed for the case of 2 states of the world.

4.1. Aggregating Guesses instead of Beliefs

In this section we discuss how our PMBA procedure can be adjusted to environments where not beliefs but rather finite guesses are elicited, e.g., which state is more likely rather than what is the probability of state $\omega = 1$ (this is also the formulation in Prelec, Seung, and McCoy (2017)—see Section 3.1). Consider a binary state space and an information structure that satisfies Assumptions 1 and 2. The action-based PMBA procedure is defined in Procedure 5.

PROCEDURE 5. Action-based PMBA, 2 states

- (1) Elicit from each agent $i \in N$, the state $\omega \in 0,1$ they consider more likely, $a_i \in 0,1$. If their belief equals 0.5, have them report $a_i = 1$.^a Calculate $\hat{\alpha}$ defined as:

$$\hat{\alpha} = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n a_i$$

- (2) Select 2 agents, A and B, such that $a_A \neq a_B$. Elicit from each of these agents $i = A, B$ their belief μ_i and their expectation of the average report in the population, i.e., denoting s_i as the signal seen by agent i , elicit for each $i = A, B$:

$$\alpha_i = \lim_{n \rightarrow \infty} \mathbb{E} \left[\frac{1}{n} \sum_{j=1}^n a_j \mid s_i \right].$$

- (3) Calculate $\bar{\alpha}(\omega_1), \bar{\alpha}(\omega_2)$ as:

$$\begin{pmatrix} \bar{\alpha}(\omega_1) \\ \bar{\alpha}(\omega_2) \end{pmatrix} = \begin{pmatrix} \mu_A & 1 - \mu_A \\ \mu_B & 1 - \mu_B \end{pmatrix}^{-1} \begin{pmatrix} \alpha_A \\ \alpha_B \end{pmatrix} \quad (6)$$

Note that the right-hand side is elicited from agents directly; and since $\alpha_A \neq \alpha_B$, the middle matrix is invertible.

- (4) Recover the state by comparing $\hat{\alpha}$ calculated in step 1 with $\bar{\alpha}(\cdot)$ calculated above. The state of the world is ω s.t. $\hat{\alpha} = \bar{\alpha}(\omega)$.

^aNote that this tie-breaking cannot be ensured by an incentive-compatible elicitation mechanism.

For the action based PMBA procedure to be viable we require the existence of two agents A, B with differing first-order guesses, $a_A \neq a_B$. Assumptions 1 and 2 are not sufficient to assure this. The following assumption, however, guarantees the existence of two such agents.

ASSUMPTION 6 (No Herding). *We assume that there exists $\delta > 0$, such that for any agent i and any state $\omega_j = \omega_1, \omega_2$, we have that the probability agent i assigns to state ω_j conditional on their signal s_i is larger than half with a probability of at least δ .*

We can now present our result on aggregation in this environment.

PROPOSITION 2. *Consider a binary state space. Suppose that the information structure satisfies Assumptions 1, and 6. Then the Action Based PMBA procedure (Procedure 5) correctly recovers the true state of the world a.s.*

The result follows along the same lines as Theorem 1.

4.2. Incentivizing Truthful Reporting

In our analysis so far we abstracted away from incentive issues, assuming truth-telling. We now discuss how truthful elicitation and aggregation can simultaneously be achieved via a slightly adjusted mechanism that includes payments which ensure truthful reporting is a (strict) Bayes Nash Equilibrium.

Recall that when there is a verifiable state that is or will be known to the principal, the principal can truthfully elicit agents' beliefs about the state by paying them using a proper scoring rule. A proper scoring rule pays agents as a function of the reported beliefs and the true state so that for any agent, truthful reporting of beliefs is a dominant strategy. For example, classic scoring rules such as the Brier score (Brier, 1950) or the logarithmic scoring rule make truthful reporting a dominant strategy for expected utility maximizing agents.¹⁷

When the state is not verifiable/observable to the principal, proper scoring rules are not applicable. However, the principal may nevertheless be interested in ensuring that agents are incentivized to report their beliefs truthfully. A leading example from Prelec (2004) is the case of subjective surveys/evaluations of a new product. In their example, the principal (e.g., a marketer) wishes to incentivize truthful reporting by agents, but there is no "ground truth" on which to base the rewards. The basic idea of Bayesian Truth Serum of Prelec (2004) is that agents' beliefs also have implications on their second-order beliefs (i.e., how they think the population will evaluate them). The population averages are indeed verifiable, and therefore can be used to incentivize agents. A similar idea works here, we describe it loosely for completeness:

- (1) Elicit beliefs and second-order beliefs from agents as prescribed in PMBA to learn the true state ω (which variant of PMBA is used, and what is elicited, depends on the setting).
- (2) For the agents who were asked for second-order beliefs, pay them based on a proper scoring rule as a function of their expected population average beliefs, and the true calculated population average beliefs.
- (3) For all other agents, pay them using a proper scoring rule as a function of their beliefs and the state ω recovered by PMBA.

¹⁷There is a large literature proposing more robust scoring rules for various settings, see e.g., Karni (2009) for an example.

Observe that under the maintained assumptions of correctness of the corresponding PMBA procedure, truthful reporting will be a perfect Bayesian equilibrium among agents. To understand why, first note that since there is a large population, the average beliefs are effectively observed: even if a single agent misreports, this does not affect the average. Therefore, rewarding the two agents with a proper scoring rule based on their expected population average beliefs is feasible and will incentivize truthful reporting. These agents have no incentive to misreport their first-order beliefs because there is no associated payment. Note that given this, the true underlying state can be correctly calculated. A proper scoring rule against this recovered state ensures the truthful reporting of first-order beliefs for all the agents. Finally, there is no “compound” incentive for an agent who reports both first- and second-order beliefs to misreport their beliefs: their incentive to report their second-order beliefs truthfully was already argued, and misreporting their first-order beliefs will cause the PMBA to fail and therefore will void payments.

5. BAYESIAN AGGREGATION WITH FIXED FINITE POPULATIONS

So far, we have discussed the possibility of aggregating information with an infinite or arbitrarily population. In practice, of course, populations are finite. The procedures given above will correctly identify the true state with high probability in large populations. For instance, in our baseline model, even with a large but finite population, the average belief in the population will concentrate around the expected belief conditional on the true state. As long as the expected beliefs conditional on state are sufficiently different across the states, an appropriately modified procedure will recover the true state of the world with high probability.

Nevertheless, at a theoretical level, one may wonder if, with a finite population, there is any value to eliciting higher-order beliefs, ignoring the difficulties of such elicitation in practice. To make this potentially valuable, we need a more exacting benchmark, since, as we argued above, PMBA will already aggregate information with appropriately high probability. The benchmark we use therefore is one of a “full information posterior,” i.e., can we, without knowledge of the underlying information structure P , and eliciting solely agents beliefs (and higher-order beliefs), nevertheless reach the same posterior beliefs as an omniscient agent who knew P and directly observed all the agents’ signals? Our previous results answered this in the positive and showed that for the case of an infinite population (under the maintained assumptions), the output of the PMBA could identify the degenerate belief on the true state.

In this section, we answer these questions primarily in the negative. First we show that with a finite population, and with elicitation of the entire hierarchy of beliefs, an agnostic procedure can learn the prior and signals of each agent. This result is essentially

a straightforward corollary of the results of [Mertens, Sorin, and Zamir \(2015\)](#). However, elicitation of the entire hierarchy is obviously impractical in real-world applications.

By contrast, we show that, for elicitation up to any finite-order of beliefs, there is an identification problem: there exist information structures where the exact same finite hierarchy can be realized among agents in both states.

Consider a finite set of agents $\{1, 2, \dots, N\}$ with a common prior P defined on a finite set $\Omega \times S$ (where $S = \times_{i=1}^N S_i$ is the set of their signal profiles). The aggregator does not know P but can ask the agents to report their higher-order beliefs. In this section, we show that the aggregator can effectively elicit the full information posterior, provided that they can ask the agents to report their entire hierarchy of beliefs and that each hierarchy of beliefs uniquely identifies a signal of an agent.

To this end, we recall the standard formulation of higher-order beliefs by [Mertens and Zamir \(1985\)](#). Denote by s_i^k the k th-order belief of agent i over Ω . For instance, $s_i^1 = \text{marg}_{\Omega} P(\cdot | s_i)$,

$$s_i^2(\omega, s_{-i}^1) = P\left(\left\{(\omega', s'_{-i}) : \omega' = \omega \text{ and } s'_{-i} = s_{-i}^1\right\} | s_i\right),$$

and so on. Denote by \bar{s}_i the hierarchy of beliefs induced by signal s_i , i.e., $\bar{s}_i = (s_i^1, s_i^2, \dots)$. Let \bar{S}_i be the set of hierarchies of beliefs induced from S_i and \bar{P} the distribution induced by P on $\Omega \times \bar{S}$. Moreover, by [Mertens and Zamir \(1985\)](#), each \bar{s}_i induces a belief $\bar{\pi}_i(\bar{s}_i)$ over $\Omega \times \bar{S}_{-i}$, where $\bar{S}_{-i} = \times_{j \neq i} \bar{S}_j$.

THEOREM 4. *Suppose that $\bar{\mu}$ has a finite support and the agents report $\bar{s} = (\bar{s}_i)_i$. Then there exists a procedure which recovers the “pooled information” posterior on the states, i.e., $\bar{P}(\cdot | \bar{s})$.*

One might ask whether such a result can be obtained while only asking agents for their higher-order beliefs upto some finite order (i.e., as opposed to the full infinite hierarchy). It is possible to show that in general, without further assumptions, this is not the case: the result of [Theorem 4](#) cannot be achieved if the aggregator only knows the reported beliefs up to order m . Our argument is adapted from the leading example of [Lipman \(2003\)](#). We defer the details to [Appendix B.1](#).

6. RELATED LITERATURE AND CONCLUSIONS

In what follows, we discuss the related literature more closely (with the benefit of being able to explain some of the ideas more formally with our notation). Finally, we conclude.

The literature on the aggregation of information in a population is perhaps far too vast to cite comprehensively. Multiple strands attempt to understand, in a variety of settings, when and how existing institutions aggregate dispersed information in strategic settings. These include studies on information aggregation in voting, a vast literature with its roots

in Condorcet, but more recently studied formally starting from Feddersen and Pesendorfer (1997); aggregation in auctions (see e.g., Milgrom and Weber (1982) and subsequent literature) and of course information aggregation in markets (see e.g., Grossman and Stiglitz (1980)). Ostrovsky (2009) proposes a market scoring rule mechanism that achieves information aggregation for some classes of securities in a dynamic market, but the payment rule there requires the state to be ex-post observable.

We limit attention here explicitly to work that studies the design/ possibility of aggregation procedures/ algorithms, or institutions that solely exist to aggregate information.

A key idea in the former space (indeed, in a sense an idea that is also at the heart of Prelec, Seung, and McCoy (2017)) is the Bayesian Truth Serum (Prelec (2004)— this is a procedure to truthfully elicit subjective information from agents by rewarding them as a function of others' reports. This paper pioneered the idea that agents' higher-order beliefs could be used to incentivize them successfully even if the planner/ designer did not know the prior among agents. A literature studying this followed. Prelec and Seung (2006) showed how to modify agents' reports to aggregate them better (akin to the prediction normalized voting rule we described in Example 1). Several extensions to the BTS have been proposed— see e.g., Cvitanić, Prelec, Riley, and Tereick (2019), Witkowski and Parkes (2012), Radanovic and Faltings (2013), and Radanovic and Faltings (2014). The latter three papers attempt to make the basic Bayesian Truth Serum robust, i.e., to ensure that it has good properties even in finite populations and to ensure that participation is individually rational, etc. More recent work has also expanded the scope of the question to continuous spaces, but with more structure—see, e.g., Kong, Schoenebeck, Tao, and Yu (2020).

These ideas have also been successfully employed in real-world applications: see e.g., Shaw, Horton, and Chen (2011) for an application on Mechanical Turk, or Rigol, Husam, and Roth (2021), who use the Robust BTS of Witkowski and Parkes (2012) to identify high-ability micro-entrepreneurs by surveying peers. There is also a growing literature attempting to elicit and use higher-order beliefs to improve aggregation—see e.g., recent works such as Palley and Soll (2019), Palley and Satopää (2020). Some of these works consider modifications of the SP algorithm that improve performance in experiments, e.g., Wilkening, Martinie, and Howe (2021) or Martinie, Wilkening, and Howe (2020). For example, Wilkening, Martinie, and Howe (2021) show that opposite to SP, SC weights forecasters with more informative private signals more than forecasters with less informative ones. This explains why SC is favorable relative to SP when both guess/vote and posterior data are available. Similarly, McCoy and Prelec (0) use Bayesian Hierarchical methods to improve performance relative to SP.

On the latter, there is an important literature which has attempted to understand the performance of prediction markets: see e.g., Wolfers and Zitzewitz (2004) for an early

piece summarizing the issues and [Baillon \(2017\)](#) for an alternate design. [Wolfers and Zitzewitz \(2006\)](#) study when the prediction market price can be interpreted as the average of traders' beliefs, while [Ottaviani and Sørensen \(2015\)](#) study when the market price under-reacts to new information. [Dai, Jia, and Kou \(2020\)](#) show how to infer the state of the world from prediction market trading data using these theoretical ideas. There is also a large literature on pari-mutuel markets, and on adapting them to aggregate information—see e.g., [Pennock \(2004\)](#).

As we mentioned in footnote 3, our approach has some similarities with Theorem 1.4 of [Prelec, Seung, and McCoy \(2017\)](#). This approach requires elicitation of $p(s_i|s_j)$, i.e. the conditional probability that a different agent has seen signal s_i given that you have seen s_j . In particular this requires that the subjects understand the meaning of signals (H and T in their example). In contrast, PMBA questions are all in terms of primitives (beliefs and higher-order beliefs) of an agent. Our position is that this distinction is important because if the subjects really understand the model/prior including meaning of signals, the easiest question is to simply ask them for the state-contingent population signal distribution (i.e. the bias of the various possible coins in the terminology of PSM).

A different approach to ours for aggregating information with more than 2 states can be found in [Libgober \(2021\)](#). The principal does not know the agent's prior, but can elicit, after an unknown Blackwell experiment, their posterior and their contingent hypothetical beliefs (their posteriors contingent on seeing any signal). He shows that the prior and experiment can be recovered from these contingent hypothetical beliefs, via a linear-regression method, even if some signals are not sampled (i.e. μ need not be a square matrix). In contrast, [Prelec and McCoy \(2022\)](#), following [Samet \(1998b\)](#), propose to identify contingent hypothetical beliefs with a transition matrix on signals. An invariant distribution of the matrix corresponds to the ex-ante population signal distribution and thereby delivers the unknown information structure. At a technical level, our "expected population average beliefs" serve a similar role to the "contingent hypothetical beliefs" in these papers. However, the contingent hypothetical beliefs include information about higher-order beliefs well beyond the expected population mean; moreover, both [Libgober \(2021\)](#) and [Prelec and McCoy \(2022\)](#) require that the agents share a common prior and thereby rule out misspecified information settings (Procedure 3).

On a related theme, there are the works of [Samet \(1998a\)](#), [Samet \(1998b\)](#), and [Golub and Morris \(2017\)](#). These papers are interested in interim characterizations of the common prior, i.e., understanding whether or not agents have a common prior from the properties of their interim beliefs/higher-order beliefs.

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APPENDIX A. PROOFS FROM SECTION 2

First we prove a general property:

LEMMA 1. *Suppose Part (2) of Assumption 1 is satisfied. Then for any state ω ,*

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^{\infty} \tilde{\mu}_i \rightarrow_P \bar{\mu}(\omega).$$

PROOF. The result is established in two steps. For any state ω below, we show that agents' beliefs that the state is $\omega' \in \Omega$ satisfy vanishing correlation. As beliefs lie in $[0, 1]$ and therefore have bounded variance, we can apply the Bernstein law of large numbers. Together, these imply that when the state of the world is ω ; for any state $\omega' \in \Omega$, we have that:

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^{\infty} \tilde{\mu}_{i,\omega'} \rightarrow_P \bar{\mu}_{\omega'}(\omega).$$

Since there are only a finite set of states, the main result follows.

First, let us prove our claim of vanishing correlations:

CLAIM 2. *Conditioned on state $\omega \in \Omega$, the sequence of (random) beliefs $\langle \tilde{\mu}_{i,\omega'} \rangle_{i=1}^{\infty}$ that the state of the world is any $\omega' \in \Omega$ satisfies vanishing correlation, i.e.,*

$$\lim_{|i-j| \rightarrow \infty} \text{cov}(\tilde{\mu}_{i,\omega'}, \tilde{\mu}_{j,\omega'} | \omega) = 0.$$

PROOF OF CLAIM. Consider agents i and j such that ε -independence among signals holds, i.e., for all events $E, E' \in \mathcal{F}$ we have

$$|P[\tilde{s}_i \in E, \tilde{s}_j \in E' | \omega] - P_{i,\omega}^S(E)P_{j,\omega}^S(E')| \leq \varepsilon.$$

Let f_i denote the conditional probability function that assigns Bayesian beliefs of state ω' to signals for agent i . By measurability of f_i 's we have that every event $S, S' \subseteq [0, 1]$ has a corresponding event $f_i^{-1}(S)$ and $f_j^{-1}(S')$ such that

$$\begin{aligned} & \left| \Pr[\tilde{\mu}_{i,\omega'} \in S, \tilde{\mu}_{j,\omega'} \in S' | \omega] - \Pr[\tilde{\mu}_{i,\omega'} \in S | \omega] \Pr[\tilde{\mu}_{j,\omega'} \in S' | \omega] \right| \\ &= \left| P[\tilde{s}_i \in f_i^{-1}(S), \tilde{s}_j \in f_j^{-1}(S') | \omega] - P_{i,\omega}^S(f_i^{-1}(S))P_{j,\omega}^S(f_j^{-1}(S')) \right| \end{aligned}$$

establishing approximate independence of beliefs, conditional on the state.

To establish state dependent vanishing correlation among beliefs, consider the random beliefs $\tilde{\mu}_{i,\omega'}, \tilde{\mu}_{j,\omega'}$. For any number $n > 1$, let G_n denote a partition of $[0, 1]$ into $n + 1$ equal intervals, i.e., $G_n^k = [\frac{k-1}{n}, \frac{k}{n})$ for $k = 1$ thru $n - 1$; $G_n^n = [\frac{n-1}{n}, 1]$.

Note that:

$$\begin{aligned} \mathbb{E}[\tilde{\mu}_{i,\omega'} | \omega] &\leq \sum_{k=1}^n \frac{k}{n} \Pr(\tilde{\mu}_{i,\omega'} \in G_n^k | \omega) \equiv U_i^n, \\ \mathbb{E}[\tilde{\mu}_{j,\omega'} | \omega] &\leq \sum_{k=1}^n \frac{k}{n} \Pr(\tilde{\mu}_{j,\omega'} \in G_n^k | \omega) \equiv U_j^n, \end{aligned}$$

$$\mathbb{E}[\tilde{\mu}_{i,\omega'} \tilde{\mu}_{j,\omega'} | \omega] \leq \sum_{l=1}^n \sum_{k=1}^n \frac{lk}{n} \Pr(\tilde{\mu}_{i,\omega'} \in G_n^l, \tilde{\mu}_{j,\omega'} \in G_n^k | \omega) \equiv U_{ij}^n.$$

Since $\tilde{\mu}_{i,\omega'}$ and $\tilde{\mu}_{j,\omega'}$ satisfy ε -independence, we have that

$$|U_{ij}^n - U_i^n U_j^n| \leq \varepsilon.$$

However, as n goes to infinity, by observation, $U_i^n \rightarrow \mathbb{E}[\tilde{\mu}_{i,\omega'} | \omega]$, $U_j^n \rightarrow \mathbb{E}[\tilde{\mu}_{j,\omega'} | \omega]$ and $U_{ij}^n \rightarrow \mathbb{E}[\tilde{\mu}_{i,\omega'} \tilde{\mu}_{j,\omega'} | \omega]$. Therefore $\text{cov}(\tilde{\mu}_{i,\omega'}, \tilde{\mu}_{j,\omega'} | \omega) \leq \varepsilon$.

Now, by part (2) of Assumption 1, we know that for any i , and $\varepsilon > 0$, $\exists n(\varepsilon)$ large enough so that if $j - i > n$ so that we have that conditional on state, $\tilde{\mu}_i$ and $\tilde{\mu}_j$ are ε -independent. The result trivially follows. \blacksquare

Now, recall the Bernstein law of large numbers (See e.g 3.3.2 of Shiryaev (2012)):

THEOREM 5. *Let \tilde{X}_i , $i = 1, 2, 3 \dots$ be a sequence of real valued random variables such that (1) $\mathbb{E}[\tilde{X}_i]$ is finite for each i , (2) there exists K finite such that $\mathbb{V}[\tilde{X}_i] \leq K$ for all i , and (3) $\lim_{|i-j| \rightarrow \infty} \text{cov}(\tilde{X}_i, \tilde{X}_j) = 0$. Then*

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n (\tilde{X}_i - \mathbb{E}[\tilde{X}_i]) \rightarrow_P 0.$$

Observe that since $\tilde{\mu}_{i,\omega'}$ is a belief, it lies between $[0, 1]$ and so the first two conditions of the Theorem are verified. Claim 2 verifies the third condition conditional on any true state ω . Therefore we have the desired result for any $\omega' \in \Omega$, conditional on true state ω . Since the set of states is finite, we have convergence in probability of the entire vector. \blacksquare

LEMMA 2. *Suppose $L = 2$. Suppose further that part (1) of Assumption 1 holds and $P_{i,\omega_1}^S \neq P_{i,\omega_2}^S$. Then following statements hold:*

- (1) $\frac{\hat{G}_{i,\omega_1}(r)}{\hat{G}_{i,\omega_2}(r)}$ is non-increasing in r
- (2) $\frac{\hat{G}_{i,\omega_1}(r)}{\hat{G}_{i,\omega_2}(r)} > 1$ for all $r \in (\beta, \bar{\beta})$ where $[\underline{\beta}, \bar{\beta}]$ denote the convex hull of the support of private beliefs.¹⁸
- (3) $\bar{\mu}_i(\omega_1) < \bar{\mu}_i(\omega_2)$.

PROOF. By Bayes' rule, we know that

$$r = \frac{dG_{i,\omega_2}(r) P(\omega_2)}{dG_{i,\omega_1}(r) P(\omega_1) + dG_{i,\omega_2}(r) P(\omega_2)}. \quad (7)$$

¹⁸Since signal generating measures are absolutely continuous each other, the support is identical across states.

We define $\alpha = P(\omega_2)/P(\omega_1)$ as the relative likelihood of the states according to the prior P . Since no signal is completely informative, i.e., $r \notin \{0, 1\}$, we can rewrite (7) as

$$\frac{dG_{i,\omega_1}}{dG_{i,\omega_2}}(r) = \left(\frac{1-r}{r}\right)\alpha. \quad (8)$$

Hence, for any $r \in (0, 1)$, we have

$$G_{i,\omega_1}(r) = \int_0^r dG_{i,\omega_1}(x) = \int_0^r \left(\frac{1-x}{x}\right)\alpha dG_{i,\omega_2}(x) \geq \left(\frac{1-r}{r}\right)\alpha G_{i,\omega_2}(r). \quad (9)$$

Part (1). Pick any r', r s.t. $r' > r$. Observe that:

$$\begin{aligned} & \frac{G_{i,\omega_1}(r)}{G_{i,\omega_2}(r)} - \frac{G_{i,\omega_1}(r')}{G_{i,\omega_2}(r')} \\ &= \frac{G_{i,\omega_1}(r)G_{i,\omega_2}(r') - G_{i,\omega_1}(r')G_{i,\omega_2}(r)}{G_{i,\omega_2}(r)G_{i,\omega_2}(r')}. \end{aligned}$$

Define $\Delta_{i,\omega} = G_{i,\omega}(r') - G_{i,\omega}(r)$. We then have:

$$= \frac{G_{i,\omega_1}(r)\Delta_{i,\omega_2} - \Delta_{i,\omega_1}G_{i,\omega_2}(r)}{G_{i,\omega_2}(r)G_{i,\omega_2}(r')},$$

Observe that $\Delta_{i,\omega_1} = \int_r^{r'} dG_{i,\omega_1}(x) = \int_r^{r'} \left(\frac{1-x}{x}\right)\alpha dG_{i,\omega_2}(x) \leq \left(\frac{1-r}{r}\right)\alpha \Delta_{i,\omega_2}$. Substituting in,

$$\begin{aligned} & \geq \frac{G_{i,\omega_1}(r)\Delta_{i,\omega_2} - \frac{1-r}{r}\alpha \Delta_{i,\omega_2}G_{i,\omega_2}(r)}{G_{i,\omega_2}(r)G_{i,\omega_2}(r')}, \\ & \geq 0, \end{aligned}$$

where the last inequality follows by substituting in $G_{i,\omega_1}(r)$ in the numerator from (9).

Part (2). It follows from (9) that

$$G_{i,\omega_1}(r) \geq G_{i,\omega_2}(r) \text{ if } \left(\frac{1-r}{r}\right)\alpha \geq 1. \quad (10)$$

If $\left(\frac{1-r}{r}\right)\alpha < 1$, then

$$\begin{aligned} 1 - G_{i,\omega_1}(r) &= \int_r^1 dG_{i,\omega_1}(x) \\ &= \int_r^1 \left(\frac{1-x}{x}\right)\alpha dG_{i,\omega_2}(x) \\ &\leq \left(\frac{1-r}{r}\right)\alpha (1 - G_{i,\omega_2}(r)) \\ &\leq 1 - G_{i,\omega_2}(r). \end{aligned} \quad (11)$$

Combining (10) and (11), we obtain that

$$\frac{G_{i,\omega_1}(r)}{G_{i,\omega_2}(r)} \geq 1. \quad (12)$$

To prove $\frac{G_{i,\omega_1}(r)}{G_{i,\omega_2}(r)} > 1$ for any $r \in (\underline{\beta}, \bar{\beta})$, suppose to the contrary that $\frac{G_{i,\omega_1}(r)}{G_{i,\omega_2}(r)} = 1$ for some $r \in (\underline{\beta}, \bar{\beta})$. We derive contradiction in each of the following two cases

Case 1: $\left(\frac{1-r}{r}\right) \alpha < 1$. Then,

$$\begin{aligned} G_{i,\omega_1}(1) &= G_{i,\omega_1}(r) + \int_r^1 dG_{i,\omega_1}(x) \\ &= G_{i,\omega_2}(r) + \int_r^1 dG_{i,\omega_1}(x) \\ &= G_{i,\omega_2}(r) + \int_r^1 \left(\frac{1-x}{x}\right) \alpha dG_{i,\omega_2}(x) \\ &\leq G_{i,\omega_2}(r) + \left(\frac{1-r}{r}\right) \alpha \int_r^1 dG_{i,\omega_2}(x) \\ &= G_{i,\omega_2}(r) + \left(\frac{1-r}{r}\right) \alpha [1 - G_{i,\omega_2}(r)] \end{aligned}$$

which yields a contradiction unless $G_{i,\omega_2}(r) = 1$. However, $G_{i,\omega_2}(r) = 1$ implies $r \geq \bar{\beta}$ which contradicts $r \in (\underline{\beta}, \bar{\beta})$.

Case 2: $\left(\frac{1-r}{r}\right) \alpha \geq 1$. Then, since $\frac{G_{i,\omega_1}(r)}{G_{i,\omega_2}(r)} = 1$ and $\frac{G_{i,\omega_1}(r)}{G_{i,\omega_2}(r)}$ is non-increasing in r , together with (12) implies that $\frac{G_{i,\omega_1}(r')}{G_{i,\omega_2}(r')} = 1$ for all $r' \geq r$. Pick $r' > \frac{\alpha}{1+\alpha}$ (note that since $\frac{\alpha}{1+\alpha}$ is just the prior belief that $\omega_1 = 1$, it must be in the interior of the support of beliefs $[\underline{\beta}, \bar{\beta}]$)—this leads to the contradiction in Case 1.

Part (3). This follows directly from the first-order stochastic dominance relationship between G_{i,ω_1} and G_{i,ω_2} in established part (2). \blacksquare

LEMMA 3. *Suppose $L = 2$ and the common prior P satisfies both Assumptions 1 and 2. Then, there exists an $\varepsilon > 0$ such that the $\bar{\mu}_i(\omega_1) + \varepsilon < \bar{\mu}_i(\omega_2)$ for all $i \in N$.*

PROOF. Fix any agent i and let the convex hull of the common support of its posteriors be $[\underline{\beta}, \bar{\beta}]$. First observe that the prior belief about the state is $\frac{\alpha}{1+\alpha}$. Therefore it must be that $\underline{\beta} < \frac{\alpha}{1+\alpha} < \bar{\beta}$ (if exactly one of these is an equality we violate the condition that the expected posterior is the prior, if both are equalities then the signal is uninformative violating Assumption 2).

CLAIM 3. For $\delta > 0$ if the total variation distance between G_{i,ω_1} and G_{i,ω_2} is larger than δ , there exists $\delta' > 0$ such that $\underline{\beta} + \delta' < \frac{\alpha}{1+\alpha} < \bar{\beta} - \delta'$, and further

$$\max \left\{ G_{i,\omega_1} \left(\frac{\alpha}{1+\alpha} - \delta' \right), (1 - G_{i,\omega_2} \left(\frac{\alpha}{1+\alpha} + \delta' \right)) \right\} \geq \delta'.$$

PROOF. To see this observe that we showed in the proof of Lemma 2 that for any $r \notin \{0, 1\}$, we have

$$\frac{dG_{i,\omega_1}}{dG_{i,\omega_2}}(r) = \left(\frac{1-r}{r} \right) \alpha.$$

Therefore for r arbitrarily close to $\frac{\alpha}{1+\alpha}$, $\frac{dG_{i,\omega_1}}{dG_{i,\omega_2}}(r)$ is arbitrarily close to 1. Further, by the fact that G_{i,ω_2} first order stochastically dominates G_{i,ω_1} , we have $\delta' > G_{i,\omega_1} \left(\frac{\alpha}{1+\alpha} - \delta' \right) > G_{i,\omega_2} \left(\frac{\alpha}{1+\alpha} - \delta' \right)$, and similarly $\delta' > (1 - G_{i,\omega_2} \left(\frac{\alpha}{1+\alpha} + \delta' \right)) > (1 - G_{i,\omega_1} \left(\frac{\alpha}{1+\alpha} + \delta' \right))$.

Therefore if the claim is not satisfied, both measures are mostly supported on $[\frac{\alpha}{1+\alpha} - \delta', \frac{\alpha}{1+\alpha} + \delta']$ the total variation distance is $\leq 2 \frac{1+\alpha}{\alpha} \delta'^2 + 2\delta'$. Therefore the claim follows for e.g., $\delta' = \frac{1}{3}\delta$. \blacksquare

Now observe that $\bar{\mu}_i(\omega) = \int_0^1 (1 - G_{i,\omega}(r)) dr$. Therefore

$$\begin{aligned} & \bar{\mu}_i(\omega_2) - \bar{\mu}_i(\omega_1) \\ &= \int_0^1 (G_{i,\omega_1}(r) - G_{i,\omega_2}(r)) dr \end{aligned}$$

We showed in Lemma 2 that G_{i,ω_2} first order stochastically dominates G_{i,ω_1} . Therefore we have, for δ' that satisfies the statement of the claim above,

$$\begin{aligned} & \geq \int_{\frac{\alpha}{1+\alpha} - \delta'}^{\frac{\alpha}{1+\alpha}} (G_{i,\omega_1}(r) - G_{i,\omega_2}(r)) dr \\ &= \delta' (G_{i,\omega_1} \left(\frac{\alpha}{1+\alpha} - \delta' \right) - G_{i,\omega_2} \left(\frac{\alpha}{1+\alpha} - \delta' \right)) \\ & \quad + \int_{\frac{\alpha}{1+\alpha} - \delta'}^{\frac{\alpha}{1+\alpha}} \left(\int_{\frac{\alpha}{1+\alpha} - \delta'}^r (dG_{i,\omega_1}(x) - dG_{i,\omega_2}(x)) \right) dr \end{aligned}$$

We know from (8) that $dG_{i,\omega_1}(r) > dG_{i,\omega_2}(r)$ for $r < \frac{\alpha}{1+\alpha}$. So we have,

$$\geq \delta' (G_{i,\omega_1} \left(\frac{\alpha}{1+\alpha} - \delta' \right) - G_{i,\omega_2} \left(\frac{\alpha}{1+\alpha} - \delta' \right)).$$

Now observe that by (9) we have

$$\begin{aligned} G_{i,\omega_1}(r) &\geq \frac{1-r}{r} \alpha G_{i,\omega_2}(r), \\ \implies G_{i,\omega_1}(r) - G_{i,\omega_2}(r) &\geq \left(\frac{1-r}{r} \alpha - 1 \right) G_{i,\omega_2}(r). \end{aligned}$$

For $r = \frac{\alpha}{1+\alpha} - \delta'$ we have

$$\implies G_{i,\omega_1}\left(\frac{\alpha}{1+\alpha} - \delta'\right) - G_{i,\omega_2}\left(\frac{\alpha}{1+\alpha} - \delta'\right) \geq \frac{\delta'(\alpha+1)^2}{\alpha - (\alpha+1)\delta'} G_{i,\omega_2}\left(\frac{\alpha}{1+\alpha} - \delta'\right).$$

However we also have by the statement of the claim that

$$\begin{aligned} G_{i,\omega_1}\left(\frac{\alpha}{1+\alpha} - \delta'\right) &\geq \delta', \\ \implies G_{i,\omega_1}\left(\frac{\alpha}{1+\alpha} - \delta'\right) - G_{i,\omega_2}\left(\frac{\alpha}{1+\alpha} - \delta'\right) &\geq \delta' - G_{i,\omega_2}\left(\frac{\alpha}{1+\alpha} - \delta'\right). \end{aligned}$$

Combining, we have that

$$\geq \max\left\{\delta' - G_{i,\omega_2}\left(\frac{\alpha}{1+\alpha} - \delta'\right), \frac{\delta'(\alpha+1)^2}{\alpha - (\alpha+1)\delta'} G_{i,\omega_2}\left(\frac{\alpha}{1+\alpha} - \delta'\right)\right\}$$

Note that the first term is decreasing in $G_{i,\omega_2}\left(\frac{\alpha}{1+\alpha} - \delta'\right)$, while the second is increasing. Therefore, the minimum is achieved at

$$\begin{aligned} \delta' - G_{i,\omega_2}\left(\frac{\alpha}{1+\alpha} - \delta'\right) &= \frac{\delta'(\alpha+1)^2}{\alpha - (\alpha+1)\delta'} G_{i,\omega_2}\left(\frac{\alpha}{1+\alpha} - \delta'\right) \\ \implies G_{i,\omega_2}\left(\frac{\alpha}{1+\alpha} - \delta'\right) &= \frac{\alpha - (\alpha+1)\delta'}{\alpha + \alpha\delta'(\alpha+1)} \delta' \end{aligned}$$

Substituting in we have that,

$$G_{i,\omega_1}\left(\frac{\alpha}{1+\alpha} - \delta'\right) - G_{i,\omega_2}\left(\frac{\alpha}{1+\alpha} - \delta'\right) \geq \frac{\delta'(\alpha+1)^2}{\alpha + \alpha\delta'(\alpha+1)} \delta',$$

and therefore we have that $\bar{\mu}_i(\omega_2) - \bar{\mu}_i(\omega_1) \geq \varepsilon$ for $\varepsilon = \frac{\delta'(\alpha+1)^2}{\alpha + \alpha\delta'(\alpha+1)} \delta'^2$. ■

We are now finally in a position to give the proof of Theorem 1.

THEOREM 1. *Suppose the information structure among agents satisfies Assumptions 1 and 2. Then the PMBA procedure (Procedure 1) correctly recovers the true state of the world almost surely.*

PROOF OF THEOREM 1. We prove this by analyzing each of the steps of Procedure 1.

(1) By Lemma 1, the law of large numbers holds and when the realized state is ω ,

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n \tilde{\mu}_i = \bar{\mu}(\omega).$$

Therefore the population mean of the elicited beliefs $\hat{\mu}$ in Step (1) of the procedure equals $\bar{\mu}(\omega)$ in state ω .

(2) By part (1) of Assumption 1, we know that the beliefs of any agent i have a common support across the states. Further, by Assumption 2, this support must have

at least 2 distinct points in it. Combining this with the fact that signals are approximately independent, there must exist at least two agents with different beliefs almost surely. So step 2 can find 2 such agents. Further, in step 2, each agent $i = A, B$ will report, by the law of total probability: $\alpha_i = \mu_i \bar{\mu}(\omega_1) + (1 - \mu_i) \bar{\mu}(\omega_2)$.

- (3) Since $\mu_A \neq \mu_B$, equation 1 can be solved.
- (4) By Lemma 3, there is a $\varepsilon > 0$ such that for all i $\bar{\mu}_i(\omega_1) < \bar{\mu}_i(\omega_2) - \varepsilon$. Therefore, the expected population average beliefs, $\bar{\mu}(\omega_1) < \bar{\mu}(\omega_2) - \varepsilon$, i.e., $\bar{\mu}(\omega_1) \neq \bar{\mu}(\omega_2)$. Therefore comparing the elicited $\hat{\mu}$ in step (1) with the recovered $(\bar{\mu}(\omega_1), \bar{\mu}(\omega_2))$ must reveal the true state. \blacksquare

THEOREM 2. *Suppose $L > 2$ and the common prior P satisfies Assumptions 1, 3, and 4. Then the PMBA procedure (Procedure 2) recovers the true unknown state of nature ω almost surely.*

PROOF. We prove this by analyzing each of the steps of Procedure 2 analogously to our proof of Theorem 1.

- (1) By Lemma 1, the law of large numbers holds and when the realized state is ω ,

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n \bar{\mu}_i = \bar{\mu}(\omega).$$

Therefore the population mean of the elicited beliefs $\hat{\mu}$ in Step (1) of the procedure equals $\bar{\mu}(\omega)$ in state ω .

- (2) By part (1) of Assumption 1, we know that the beliefs of any agent i have a common support across the states. Further, by Assumption 4, this support must have an interior relative to Δ^L . Combining this with the fact that signals are approximately independent, there must exist at least L agents whose beliefs form a full rank matrix a.s.. So step 2 can find L such agents. Further, in step 2, each agent $i \in I$ will report, by the law of total probability: $\alpha_i = \mu_i^T \bar{\mu}$.
- (3) Since the set I of agents is selected so that μ is full-rank, equation 2 can be solved.
- (4) By Assumption 3, $\bar{\mu}(\omega) \neq \bar{\mu}(\omega')$ for any $\omega, \omega' \in \Omega$. Therefore comparing the elicited $\hat{\mu}$ in step (1) with the recovered $\bar{\mu}$ must reveal the true state. \blacksquare

CLAIM 1. *Suppose that signals are conditionally i.i.d. draws from a finite set S , and Assumption 3 is satisfied. If each agent gets $L - 1$ independent draws from the distribution P_ω^S , then Assumption 4 is satisfied with the signal space S^{L-1} .*

PROOF. At each ω , the distribution of the $L - 1$ signals can be summarized by the outer product

$$\left(\otimes P_\omega^S \right)^{L-1} = \underbrace{P_\omega^S \otimes \cdots \otimes P_\omega^S}_{L-1 \text{ times}}$$

Since Assumption 3 is satisfied, the dimension of the linear space spanned by $\{P_\omega^S\}_{\omega \in \Omega}$ is at least 2. Hence, by Lemma 4 by Fu, Haghpanah, Hartline, and Kleinberg (2021), $\{(\otimes P_\omega^S)^{L-1}\}_{\omega \in \Omega}$ are linearly independent and hence $\{(\otimes P_\omega^S)^{L-1} P(\omega)\}_{\omega \in \Omega}$ are also linearly independent ($P(\omega)$ denotes the prior probability of ω). Identify $\{(\otimes P_\omega^S)^{L-1} P(\omega)\}_{\omega \in \Omega}$ with a $|S|^{L-1} \times L$ matrix which we denote by M . Hence, $\text{rank}(M) = L$. For $\mathbf{s} = (s^1, \dots, s^{L-1})$ and $\omega \in \Omega$, observe that $M_{\mathbf{s}, \omega} = P(\mathbf{s}, \omega)$. Since $\text{rank}(M) = L$, the conditional probability matrix $\{P(\omega|\mathbf{s})\}$ also has rank L . Hence, Assumption 4 is satisfied with the signal space S^{L-1} . ■

A.1. Proof of Theorem 3

We first need some more lemmata.

LEMMA 4. Let $\bar{\mu}_A, \bar{\mu}_B$ denote the random group mean beliefs and μ_A, μ_B their expectation. We have that $\bar{\mu}_A$ (respectively, $\bar{\mu}_B$) converges in probability to the expectation μ_A (respectively, μ_B).

PROOF. Let x_{iA}, x_{iB} denote the indicator random variable of i belonging to group A, B , and let μ_i denote the random belief of agent i . We have that

$$\bar{\mu}_A = \lim_{n \in \mathbb{N}} \frac{1}{\sum_{i < n} x_{iA}} \sum_{i < n} x_{iA} \mu_i = \lim_{n \in \mathbb{N}} \frac{1}{\frac{1}{n} \sum_{i < n} x_{iA}} \frac{1}{n} \sum_{i < n} x_{iA} \mu_i.$$

Let

$$L_n = \frac{1}{\frac{1}{n} \sum_{i < n} x_{iA}}, \text{ and, } K_n = \frac{1}{n} \sum_{i < n} x_{iA} \mu_i.$$

Note that since the conditions for Bernstein's LLN are satisfied for the sequences $\langle x_{iA} \rangle$ and $\langle x_{iA} \mu_i \rangle$ conditional on the realized state, it follows that $\langle L_n \rangle$ and $\langle K_n \rangle$ converge in probability to L respectively K where

$$L = \lim_{n \in \mathbb{N}} E[L_n], \quad K = \lim_{n \in \mathbb{N}} E[K_n].$$

Since $\langle L_n \rangle$ and $\langle K_n \rangle$ are uniformly bounded, it follows that $\bar{\mu}_A$ converges in probability to μ_A

$$\mu_A = \lim_{n \in \mathbb{N}} \frac{1}{E\left[\frac{1}{n} \sum_{i < n} x_{iA}\right]} E\left[\frac{1}{n} \sum_{i < n} x_{iA} \mu_i\right],$$

and analogously $\bar{\mu}_B$ converges in probability to μ_B . ■

LEMMA 5. Let $\tilde{\alpha}_A, \tilde{\alpha}_B$ denote the groups random population beliefs. We have that $\tilde{\alpha}_A, \tilde{\alpha}_B$ converge in probability to

$$\mu_A \hat{\mu}^1 + (1 - \mu_A) \hat{\mu}^2, \mu_B \hat{\mu}^1 + (1 - \mu_B) \hat{\mu}^2,$$

where $\hat{\mu}^1 = \bar{\mu}(\omega_1) + E[\zeta^1]$, $\hat{\mu}^2 = \bar{\mu}(\omega_2) + E[\zeta^2]$.

PROOF. Note that

$$\alpha^A = \lim_{n \in \mathbb{N}} \frac{1}{\frac{1}{n} \sum_{i < n} x_{iA}} \frac{1}{n} \sum_{i < n} x_{iA} \left(\mu_i \bar{\mu}(\omega_1) + (1 - \mu_i) \bar{\mu}(\omega_2) + \mu_i \zeta_i^1 + (1 - \mu_i) \zeta_i^2 \right).$$

By Lemma 4, we have that:

$$\frac{1}{\frac{1}{n} \sum_{i < n} x_{iA}} \frac{1}{n} \sum_{i < n} x_{iA} (\mu_i \bar{\mu}(\omega_1) + (1 - \mu_i) \bar{\mu}(\omega_2)) \rightarrow_P \mu_A \bar{\mu}(\omega_1) + (1 - \mu_A) \bar{\mu}(\omega_2).$$

For the random term

$$\frac{1}{\frac{1}{n} \sum_{i < n} x_{iA}} \frac{1}{n} \sum_{i < n} x_{iA} (\mu_i \zeta_i^1 + (1 - \mu_i) \zeta_i^2)$$

let, $L_n = \frac{1}{\frac{1}{n} \sum_{i < n} x_{iA}}$, and, $\hat{K}_n = \frac{1}{n} \sum_{i < n} x_{iA} (\mu_i \zeta_i^1 + (1 - \mu_i) \zeta_i^2)$.

Since the sequences of random variables $\langle x_{iA} \rangle$, $\langle x_{iA} \mu_i \zeta_i^1 \rangle$ and $\langle x_{iA} \mu_i \zeta_i^2 \rangle$ satisfy the conditions of the Bernstein LLN (conditional on the state ω), it follows that $\langle L_n \rangle$ and $\langle \hat{K}_n \rangle$ converge in probability to L respectively \hat{K} where

$$L = \lim_{i \in \mathbb{N}} E \left[\frac{1}{\frac{1}{n} \sum_{i < n} x_{iA}} \right] \text{ and, } \hat{K} = \lim_{i \in \mathbb{N}} \frac{1}{n} \sum_{i < n} E \left[x_{iA} \mu_i \zeta_i^1 \right] + E \left[x_{iA} (1 - \mu_i) \zeta_i^2 \right].$$

By independence of error terms and first-order beliefs this equals it follows that

$$\hat{K} = \lim_{i \in \mathbb{N}} E[\zeta^1] \left(\frac{1}{n} \sum_{i < n} E[x_{iA} \mu_i] \right) + E[\zeta^2] \left(\frac{1}{n} \sum_{i < n} E[x_{iA} (1 - \mu_i)] \right).$$

Since $\langle L_n \rangle$ and $\langle \hat{K}_n \rangle$ are uniformly bounded, and by Lemma 4 it follows that the random term

$$\frac{1}{\frac{1}{n} \sum_{i < n} x_{iA}} \frac{1}{n} \sum_{i < n} x_{iA} (\mu_i \zeta_i^1 + (1 - \mu_i) \zeta_i^2)$$

converges in probability, and the claim follows, i.e., $\tilde{\alpha}^A$ converges in probability to

$$\mu_A E[\zeta^1] + (1 - \mu_A) E[\zeta^2]$$

concluding the proof. ■

We are now in a position to provide a proof of the main theorem, Theorem 3 (restated below for the reader's convenience).

THEOREM 3. *Suppose the true signal distribution process satisfies Assumptions 1 and 2, and that the agents' knowledge regarding the information structure satisfies Assumption 5. Suppose further that*

$$\left|E[\zeta^1]\right| + \left|E[\zeta^2]\right| < \bar{\mu}(\omega_1) - \bar{\mu}(\omega_2). \quad (5)$$

Then, Misspecified Information Population-Mean-Based Aggregation (Procedure 3) correctly recovers the true state of the world almost surely.

PROOF. From above, we have that

$$\begin{aligned} \hat{\mu}^1 &= \bar{\mu}(\omega_1) + E[\zeta^1] \\ \hat{\mu}^2 &= \bar{\mu}(\omega_2) + E[\zeta^2]. \end{aligned}$$

We need to show that

$$|\hat{\mu}^\omega - \bar{\mu}(\omega)| < |\hat{\mu}^{-\omega} - \bar{\mu}(\omega)|$$

for $\omega = \omega_1, \omega_2$. Consider the case of ω_1 . Note that by Lemma 5 we have

$$\left|\hat{\mu}^1 - \bar{\mu}(\omega_1)\right| = \left|E[\zeta^1]\right|.$$

For the correct identification of state ω_1 we thus require that

$$\left|\hat{\mu}^2 - \bar{\mu}(\omega_1)\right| > \left|E[\zeta^1]\right|.$$

By Lemma 5 it follows that

$$\left|\hat{\mu}^2 - \bar{\mu}(\omega_1)\right| = \left|\bar{\mu}(\omega_1) - \bar{\mu}(\omega_2) - E[\zeta^2]\right|. (*)$$

Assuming that

$$\left|E[\zeta^2]\right| + \left|E[\zeta^1]\right| < \bar{\mu}(\omega_1) - \bar{\mu}(\omega_2)$$

and plugging into equation (*) yields

$$\left|\hat{\mu}^2 - \bar{\mu}(\omega_1)\right| > \left|\left|E[\zeta^2]\right| + \left|E[\zeta^1]\right| - E[\zeta^2]\right| \geq \left|E[\zeta^1]\right|$$

which establishes correct identification of state ω_1 . The argument for the correct identification of state ω_2 is analogous. \blacksquare

APPENDIX B. PROOFS FROM SECTION 5

THEOREM 4. *Suppose that $\bar{\mu}$ has a finite support and the agents report $\bar{s} = (\bar{s}_i)_i$. Then there exists a procedure which recovers the "pooled information" posterior on the states, i.e., $\tilde{P}(\cdot|\bar{s})$.*

PROOF. Assume that $\text{marg}_{\bar{s}} \tilde{\mu}(\bar{s}) > 0$. It follows from Theorem III.2.7 of Mertens, Sorin, and Zamir (2015) that the aggregator can derive:

- (1) The set $E(\bar{s})$ which is the smallest set $Y \subseteq \Omega \times \bar{S}$ of state-hierarchy profiles satisfying
 - (a) $(\omega, \bar{s}) \in Y$ for some $\omega \in \Omega$, and,
 - (b) for each $(\omega, \bar{t}) \in Y$, we have $\{\bar{t}_i\} \times \text{supp } \tilde{\pi}_i(\bar{t}_i) \subseteq Y$.
- (2) The unique consistent probability $\tilde{\mu}$ on $E(\bar{s})$.

With (1) and (2), the aggregator can compute $\tilde{\mu}(\cdot|\bar{s})$. We recap and illustrate both (1) and (2) for the ease of reference.

To construct (1), define $C_i^1(\omega, \bar{t}) = \{\bar{t}_i\} \times \text{supp } \tilde{\pi}_i(\bar{t}_i)$ for each $(\omega, \bar{t}) \in \Omega \times \bar{S}$; and inductively, for every $l \geq 1$ and $\bar{t} \in \bar{S}$, define

$$C_i^{l+1}(\omega, \bar{t}) = C_i^l(\omega, \bar{t}) \cup \bigcup_{(\omega', \bar{t}') \in C_i^l(\omega, \bar{t})} \bigcup_{j=1}^N C_j^1(\omega', \bar{t}').$$

Then, let

$$C_i(\omega, \bar{t}) = \bigcup_{l=1}^{\infty} C_i^l(\omega, \bar{t}).$$

These include (ω, \bar{t}) , the state-hierarchy profiles which agent i regards as possible (i.e., $C_i^1(\omega, \bar{t})$) at (ω, \bar{t}) , the state-hierarchy profiles which some agent regards at some state-hierarchy profile in $C_i^1(\omega, \bar{t})$ (i.e., $C_i^2(\omega, \bar{t})$), and so on.

Consider any ω^* such that $\tilde{\mu}(\omega^*, \bar{s}) > 0$. Note that $C_i(\omega^*, \bar{s})$ satisfies properties (a) and (b) above by construction and hence $C_i(\omega^*, \bar{s}) \supseteq E(\bar{s})$.¹⁹ Also we can argue that $C_i(\omega^*, \bar{s}) \subseteq E(\bar{s})$ inductively.

For (2), it follows from Bayes' rule that for every $(\omega', \bar{t}') \in C_i^1(\omega, \bar{t})$ and $\tilde{\mu}(\omega, \bar{t}) > 0$, we must have $\bar{t}_i = \bar{t}'_i$ and hence

$$\frac{\tilde{\mu}(\omega', \bar{t}')}{\tilde{\mu}(\omega, \bar{t})} = \frac{\tilde{\pi}_i(\bar{t}'_i)(\omega', \bar{t}') \tilde{\mu}_i(\bar{t}'_i)}{\tilde{\pi}_i(\bar{t}_i)(\omega, \bar{t}_{-i}) \tilde{\mu}_i(\bar{t}_i)} = \frac{\tilde{\pi}_i(\bar{t}'_i)(\omega', \bar{t}')}{\tilde{\pi}_i(\bar{t}_i)(\omega, \bar{t}_{-i})} > 0.$$

The aggregator knows $\tilde{\pi}_i(\bar{t}_i)$ and hence can express $\tilde{\mu}(\omega', \bar{t}')$ as a multiple of $\tilde{\mu}(\omega, \bar{t})$. Inductively, for every $(\omega', \bar{t}') \in C_i^l(\omega, \bar{t})$, $\tilde{\mu}(\omega', \bar{t}')$ can also be expressed as a multiple of $\tilde{\mu}(\omega, \bar{t})$.

Hence, $\tilde{\mu}$ is uniquely determined on $E(\bar{s}) = C_i(\omega^*, \bar{s})$ since $\tilde{\mu}(\omega^*, \bar{s}) > 0$. $\tilde{\mu}(\cdot|\bar{s})$ can thus be calculated. \blacksquare

¹⁹Property (b) is by construction and property (a) follows from the assumption that $\tilde{\mu}(\omega^*, \bar{s}) > 0$.

B.1. *Eliciting Higher-Order Beliefs up to a Finite Order m*

We adopt his notation here for ease of comparison. Suppose there are two players and two states of nature $\{\omega_1, \omega_2\}$ as in our paper. The construction is easier to explain in terms of the standard partitional model of knowledge.

The model below considers 8 extended states: $\{(\sigma_l, k) : l \in \{1, 2\} \text{ and } k \in \{1, 2, 3, 4\}\}$. Interpret state (σ_l, k) in this model as corresponding to ω_l realized for $l = 1, 2$ and any k . In this example, Lipman considers the hierarchy of belief induced by common knowledge that player 1 assigns probability $2/3$ to ω_1 and player 2 assigns probability $1/3$ to ω_1 . Such a type does not admit a common prior. However, Lipman shows that the model below, which has a uniform common prior, admits a state $(\sigma_1, 1)$ where the players have the same belief as that of the common knowledge type described above, up to any finite order m .

The prior for the players is the uniform distribution over the extended states, i.e.:

| | | | | | | | | |
|-------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| | $(\sigma_1, 4)$ | $(\sigma_1, 3)$ | $(\sigma_2, 2)$ | $(\sigma_1, 1)$ | $(\sigma_2, 1)$ | $(\sigma_1, 2)$ | $(\sigma_2, 3)$ | $(\sigma_2, 4)$ |
| μ | $\frac{1}{8}$ |

Agents' information is identified with the following partitions:

$$\Pi_1 = \{ \{(\sigma_1, 4), (\sigma_1, 3), (\sigma_2, 2)\} \{(\sigma_1, 1), (\sigma_2, 1), (\sigma_1, 2)\} \{(\sigma_2, 3), (\sigma_2, 4)\} \}$$

$$\Pi_2 = \{ \{(\sigma_1, 4), (\sigma_1, 3)\} \{(\sigma_2, 2), (\sigma_1, 1), (\sigma_2, 1)\} \{(\sigma_1, 2), (\sigma_2, 3), (\sigma_2, 4)\} \}.$$

Note that each $\Pi_i(\sigma', k)$ is identified with a signal of player i . Observe that player 1 assigns probability one to ω_2 at $\Pi_1(\sigma_2, 4)$ and player 2 assigns probability one to ω_1 at $\Pi_2(\sigma_1, 4)$. Hence, each partition cell of each player induces a different second-order belief. In particular, $(\Pi_1(\sigma_1, 1), \Pi_2(\sigma_1, 1))$ is the only partition profile at which the second-order beliefs of both players are identical to t . Moreover, conditional on the reported second-order belief at $(\Pi_1(\sigma_1, 1), \Pi_2(\sigma_1, 1))$ (denoted as $(\Pi_1(\sigma_1, 1), \Pi_2(\sigma_1, 1))^2$), we have

$$\mu(\omega_1 | (\Pi_1(\sigma_1, 1), \Pi_2(\sigma_1, 1))) = \frac{1}{2}.$$

Now consider another model modified from the previous one by adding one additional state with the following prior:

| | | | | | | | | | |
|--------|------------------|------------------|------------------|------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| | $(\sigma_1, 4)'$ | $(\sigma_1, 3)'$ | $(\sigma_2, 2)'$ | $(\sigma_1, 1)'$ | $(\sigma_1, 1)$ | $(\sigma_2, 1)$ | $(\sigma_1, 2)$ | $(\sigma_2, 3)$ | $(\sigma_2, 4)$ |
| μ' | $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | 0 | $\frac{1}{10}$ | $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ |

where $'$ is to indicate that a state is to the left of $(\sigma_1, 1)$ and it will be useful in generalizing the idea to eliciting m orders of beliefs, for $m \geq 3$ in what follows. Agents' information is now given by the partitions:

$$\Pi_1' = \{ \{(\sigma_1, 4)', (\sigma_1, 3)', (\sigma_2, 2)', (\sigma_1, 1)'\} \{(\sigma_1, 1), (\sigma_2, 1), (\sigma_1, 2)\} \{(\sigma_2, 3), (\sigma_2, 4)\} \};$$

$$\Pi'_2 = \{ \{ (\sigma_1, 4)', (\sigma_1, 3)' \}, \{ (\sigma_2, 2)', (\sigma_1, 1)', (\sigma_1, 1), (\sigma_2, 1) \}, \{ (\sigma_1, 2), (\sigma_2, 3), (\sigma_2, 4) \} \}.$$

Observe now that, conditional on the reported second-order belief at $(\Pi'_1(\sigma_1, 1), \Pi'_2(\sigma_1, 1))$ (denoted as $(\Pi'_1(\sigma_1, 1), \Pi'_2(\sigma_1, 1))^2$), we have

$$\mu' \left(\omega_1 \mid (\Pi'_1(\sigma_1, 1), \Pi'_2(\sigma_1, 1))^2 \right) = 0.$$

Second, observe that

$$(\Pi'_1(\sigma_1, 1), \Pi'_2(\sigma_1, 1))^2 = (\Pi_1(\sigma_1, 1), \Pi_2(\sigma_1, 1))^2.$$

This means that the reported second-order beliefs at $(\Pi'_1(\sigma_1, 1), \Pi'_2(\sigma_1, 1))$ under μ' are the same as at $(\Pi_1(\sigma_1, 1), \Pi_2(\sigma_1, 1))$ under μ . The basic idea is that in μ' , we “shift” the probability which μ assigns to $(\sigma_1, 1)$ to the additional state $(\sigma_1, 1)'$. This additional state helps us preserve the first-order belief of player 2 at $\Pi'_2(\sigma_1, 1)$ at $1/3$, while we decrease the probability $(\sigma_1, 4)$ to 0 to preserve the first-order belief of player 1 at $\Pi'_1((\sigma_1, 4)')$. This takes care of the states to the left of $(\sigma_1, 1)$. For states to the right, we double the probability of $(\sigma_2, 2)$, $(\sigma_2, 3)$, and $(\sigma_2, 4)$, so that the first-order belief of player 1 at $\Pi'_1(\sigma_1, 1)$ and the first-order belief of player 2 at Π' are also preserved. Therefore, just eliciting the first two orders of beliefs, we cannot distinguish between the model corresponding to μ (under which the posterior would be that both states are equally likely), and μ' (under which the posterior would be that the state is ω_2 for sure).

The construction for $m > 2$ is similar but more involved, see below.

B.2. Construction for $m > 2$

Similarly, we can generalize the construction in Section B.1 to the case with $m \geq 3$ as follows:

$$\begin{aligned} \Pi_1 &= \left\{ \left\{ \{ (\sigma_1, 2k-1), (\sigma_1, 2k), (\sigma_2, k) \} : k = 1, \dots, 2^{m-1} \right\}, \left\{ (\sigma_2, 2^{m-1} + 1), \dots, (\sigma_2, 2^m) \right\} \right\}; \\ \Pi_2 &= \left\{ \left\{ \{ (\sigma_2, 2k-1), (\sigma_2, 2k), (\sigma_1, k) \} : k = 1, \dots, 2^{m-1} \right\}, \left\{ (\sigma_1, 2^{m-1} + 1), \dots, (\sigma_1, 2^m) \right\} \right\}. \end{aligned}$$

$$\mu(\sigma_l, k) = \frac{1}{2^{m+1}}, \forall l = 1, 2, \forall k = 1, 2, \dots, 2^m.$$

Without loss of generality, assume that $m \geq 3$ is odd. Construct the new model as follows:

$$\Pi'_1 = \left\{ \begin{array}{l} \{ (\sigma_1, 1), (\sigma_2, 1), (\sigma_1, 2) \}, \{ (\sigma_1, 1)', (\sigma_2, 2)', (\sigma_1, 3)', (\sigma_1, 4)' \}, \\ \left\{ \left\{ (\sigma_1, 2k-1)', (\sigma_1, 2k)', (\sigma_2, k)' \right\} : k = 2^{n-1} + 1, \dots, 2^n \text{ and } n = 3, 5, \dots, m-2 \right\}, \\ \left\{ \left\{ (\sigma_1, 2k-1), (\sigma_1, 2k), (\sigma_2, k) \right\} : k = 2^{n-1} + 1, \dots, 2^n \text{ and } n = 2, 4, \dots, m-1 \right\}, \\ \left\{ (\sigma_2, 2^{m-1} + 1)', \dots, (\sigma_2, 2^m)' \right\} \end{array} \right\};$$

$$\Pi'_2 = \left\{ \begin{array}{l} \{(\sigma_1, 1), (\sigma_2, 1), (\sigma_1, 1)', (\sigma_2, 2)'\}, \\ \{ \{(\sigma_2, 2k-1)', (\sigma_2, 2k)', (\sigma_1, k)'\} : k = 2^{n-1} + 1, \dots, 2^n \text{ and } n = 2, 4, \dots, m-1 \}, \\ \{ \{(\sigma_2, 2k-1), (\sigma_2, 2k), (\sigma_1, k)\} : k = 2^{n-1} + 1, \dots, 2^n \text{ and } n = 1, 3, \dots, m-2 \}, \\ \{(\sigma_1, 2^{m-1} + 1), \dots, (\sigma_1, 2^m)\} \end{array} \right\}.$$

As in the case of $m = 2$, we start from the partition cells $\{(\sigma_1, 1), (\sigma_2, 1), (\sigma_1, 2)\}$ and $\{(\sigma_1, 1), (\sigma_2, 1), (\sigma_1, 1)', (\sigma_2, 2)'\}$ which contain $(\sigma_1, 1)$. The states with $'$ are those "to the left" of $(\sigma_1, 1)$, whereas the states without $'$ are like those "to the right" of $(\sigma_1, 1)$. We can then mimic the idea for $m = 2$ to solve for a prior μ' with the desired properties as follows:

- (1) Again, for $x > 0$, set

$$\mu'(\sigma_1, 1) = 0 \text{ and } \mu'(\sigma_2, 1) = \mu'(\sigma_1, 1)' = \mu'(\sigma_2, 2)' = x.$$

- (2) The number of states with $'$ excluding $(\sigma_1, 1)'$ and $(\sigma_2, 2)'$ (i.e., states in $\Pi_2(\sigma_1, k)'$ where $k = 2^{n-1} + 1, \dots, 2^n$ and $n = 2, 4, \dots, m-1$) is :

$$y \equiv 3 \times (2^1 + 2^3 + \dots + 2^{m-2}),$$

i.e., $y = 2^m - 2$.

We assign probability $\frac{x}{2}$ to each of these "left" states.

- (3) The number of states without $'$ excluding $(\sigma_1, 1)$ and $(\sigma_2, 1)$ (i.e., $(\sigma_2, 2)$ and states in $\Pi_1(\sigma_2, k)$ where $k = 2^{n-1} + 1, \dots, 2^n$ and $n = 2, 4, \dots, m-1$) is:

$$1 + 3 \times (2^1 + 2^3 + \dots + 2^{m-2}) = y + 1.$$

We assign probability $2x$ to each of the "right" states.

- (4) Hence for the total probability to sum to 1, we must have:

$$\begin{aligned} 3x + y\frac{x}{2} + (y+1)2x &= 1, \\ \implies x &= \frac{2}{10 + 5y}, \\ \implies x &= \frac{1}{5 \times 2^m}. \end{aligned}$$

In summary, observe that as desired, we have that

$$\begin{aligned} \mu(\omega_1 | (\Pi_1(\sigma_1, 1), \Pi_2(\sigma_1, 1))^m) &= \frac{1}{2}. \\ \mu'(\omega_1 | (\Pi'_1(\sigma_1, 1), \Pi'_2(\sigma_1, 1))^m) &= 0. \\ (\Pi'_1(\sigma_1, 1), \Pi'_2(\sigma_1, 1))^m &= (\Pi_1(\sigma_1, 1), \Pi_2(\sigma_1, 1))^m. \end{aligned}$$

Clearly, we can flip left and right to construct another model Π_1'' and Π_2'' with

$$\begin{aligned} \mu' \left(\omega_1 \mid (\Pi_1''(\sigma_1, 1), \Pi_2''(\sigma_1, 1))^m \right) &= 1; \\ (\Pi_1''(\sigma_1, 1), \Pi_2''(\sigma_1, 1))^m &= (\Pi_1(\sigma_1, 1), \Pi_2(\sigma_1, 1))^m. \end{aligned}$$

The construction here makes use of the feature that all higher-order beliefs of the common knowledge hierarchy t are degenerate. More precisely, this feature ensures that in “shifting” out the probability which μ assigns to $(\sigma_1, 1)$, we can preserve the higher-order beliefs of $\Pi_1(\sigma_1, 1)$ and $\Pi_2(\sigma_1, 1)$ as long as we can preserve the first-order belief at every partition cell except for $\Pi_1(\sigma_2, 2^m)$ and $\Pi_2(\sigma_1, 2^m)$.

APPENDIX C. COUNTEREXAMPLES TO SP/ SC FOR MULTIPLE STATES

EXAMPLE 1. Suppose there are three states $\Omega = \{\omega_1, \omega_2, \omega_3\}$, and three signals $S = \{s_1, s_2, s_3\}$. All agents have the initial uniform prior. Agents have a uniform prior over signals and receive conditionally i.i.d. signals, conditional on the state, so that their posteriors can be described as:

| | $P(\omega_1 \cdot)$ | $P(\omega_2 \cdot)$ | $P(\omega_3 \cdot)$ |
|-------|---------------------|---------------------|---------------------|
| s_1 | 0.4 | 0.21 | 0.39 |
| s_2 | 0.45 | 0.54 | 0.01 |
| s_3 | 0.44 | 0.06 | 0.5 |

i.e., the i^{th} row and j^{th} column is the posterior the agent places on ω_j upon signal s_i , $P(\omega_j|s_i)$. Note that this violates the assumption of “diagonal dominance”— $P(\omega_1|s_2) > P(\omega_1|s_1)$, i.e., an agent who sees s_2 (or, indeed s_3) places a higher posterior on ω_1 than an agent who sees signal s_1 . However note that an agent who sees signal s_i believes that ω_i is the most likely state.

To be clear, note that this can be achieved by the following distribution of signals conditional on the state (numbers rounded):

| | $P(\cdot \omega_1)$ | $P(\cdot \omega_2)$ | $P(\cdot \omega_3)$ |
|-------|---------------------|---------------------|---------------------|
| s_1 | 0.31 | 0.259 | 0.433 |
| s_2 | 0.349 | 0.667 | 0.011 |
| s_3 | 0.341 | 0.074 | 0.556 |

This results, by mechanical calculation, on the following average beliefs in the population:

| | $\bar{\mu}(\omega_1)$ | $\bar{\mu}(\omega_2)$ | $\bar{\mu}(\omega_3)$ |
|------------|-----------------------|-----------------------|-----------------------|
| ω_1 | 0.431 | 0.436 | 0.422 |
| ω_2 | 0.274 | 0.419 | 0.130 |
| ω_3 | 0.295 | 0.145 | 0.447 |

here the i^{th} row and j^{th} column represent the average belief in the population that the state is ω_i when the true state is ω_j .

Note that the resulting expected population average belief is:

| | $\alpha(s_1)$ | $\alpha(s_2)$ | $\alpha(s_3)$ |
|------------|---------------|---------------|---------------|
| ω_1 | 0.429 | 0.434 | 0.427 |
| ω_2 | 0.248 | 0.351 | 0.211 |
| ω_3 | 0.323 | 0.215 | 0.362 |

i.e., the column labeled $\alpha(s_i)$ is the expected population average belief of an agent seeing signal s_i .

More generally, we can summarize the set of “surprisingly popular” state(s) in each case as:

| | ω_1 | ω_2 | ω_3 |
|---------------|----------------------------------|----------------------------------|------------|
| $\alpha(s_1)$ | $\omega_1, \underline{\omega_2}$ | $\omega_1, \underline{\omega_2}$ | ω_3 |
| $\alpha(s_2)$ | ω_3 | $\omega_1, \underline{\omega_2}$ | ω_3 |
| $\alpha(s_3)$ | $\omega_1, \underline{\omega_2}$ | $\omega_1, \underline{\omega_2}$ | ω_3 |

i.e., the entry in row i corresponding to $\alpha(s_i)$ and column j corresponding to state ω_j is the set of states that are surprisingly popular, in true state ω_j , relative to the expected population average beliefs of an agent who received signal s_i . The underlined entry, if there are multiple, is the one which is surprising by the largest magnitude. For example, relative to an agent seeing s_1 in true state ω_1 , both ω_1 and ω_2 are surprisingly popular, but ω_2 is the most surprisingly popular: $(0.274 - 0.248 = 0.028 > 0.02 = 0.431 - 0.429)$.

Note that in our example, the “most surprisingly popular” procedure fails to identify the true state when the state is ω_1 , regardless of the signal of who is polled for their expectation about the population average beliefs. Indeed, if an agent who saw signal s_2 is polled, the true state is not even in the set of surprisingly popular states. Note also the failure of first-order stochastic dominance of population average beliefs in this 3-state example—as we argued earlier, that was key to why SP works with binary states, but this can be violated with three or more states.

Another closely related approach proposed is that of “prediction normalized votes” (see Section 1.3 of the Supplementary appendix of Prelec, Seung, and McCoy (2017)): each agent votes for the state they believe is more likely (which, in this example, was the signal they saw, i.e., an agent votes for state ω_i if they see signal s_i). The fraction of votes each state ω_j receives is normalized by the sum, over all states ω_k , the ratio of predicted vote fraction for ω_k by a ω_j voter to the predicted vote fraction for ω_j by an ω_k voter. A simple calculation shows that given the specific numerical assumption we constructed, ω_2 will have a higher prediction normalized vote than ω_1 when the true state is ω_1 .

Academic Session Schedule

Academic Session I

Date: JUN 13rd, 2025 (Friday)

Venue: College of Management, CCU

| |
|--------------------------------------|
| SESSION 1-1 ESG and green finance |
|--------------------------------------|

| |
|---------------------------|
| 10:05 – 11:05 Room 102 |
|---------------------------|

Session Chair: **Min-Hung Tsay** 蔡明宏 (Academia Sinica)

How Does ESG Rating Uncertainty Affect Corporate Capital Structure?

Presenter: Yu-Chen Wei 魏裕珍 (National Kaohsiung University of Science and Technology)

Discussant: Yu-han Lin 林語涵 (National Sun Yat-sen University)

Institutional Investor Stewardship and GHG Emissions Reduction : Evidence from Taiwan

Presenter: Min-Hung Tsay 蔡明宏 (Academia Sinica)

Discussant: Pei-Jie Hsiao 蕭珮婕 (National Sun Yat-sen University)

ESG Performance and Stock Returns: How Investor Sentiment and Attention Drive Delayed Returns in Chinese Markets

Presenter: Yu-han Lin 林語涵 (National Sun Yat-sen University)

Discussant: Shiang-Ting Tseng 曾湘婷 (National Chengchi University)

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|---|
| SESSION 1-2 Corporate finance and governance |
|---|

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|---------------------------|
| 10:05 – 11:05 Room 103 |
|---------------------------|

Session Chair: **Po-Hsin Ho** 何柏欣 (National Central University)

CEO Gender and Firm Risks: The Impact of Compensation Incentives

Presenter: Chia-Hsien Lin 林佳賢 (National Chung Cheng University)

Discussant: Mei Hsu 徐美 (National Taiwan Normal University)

Co-opted boards and Directors' and Officers' Liability Insurance

Presenter: Vivian Tai 戴維芯 (National Chi Nan University)

Discussant: Chia-Hsien Lin 林佳賢 (National Chung Cheng University)

Detailed Decomposition on Earnings Management Distribution –The Unconditional Quantile Regression Analyses

Presenter: Mei Hsu 徐美 (National Taiwan Normal University)

Discussant: Vivian Tai 戴維芯 (National Chi Nan University)

Academic Session I

Date: JUN 13rd, 2025 (Friday)

Venue: College of Management, CCU

| | |
|--------------------------------------|---------------------------|
| SESSION 1-3 ESG and green finance | 10:05 – 11:05 Room 105 |
|--------------------------------------|---------------------------|

Session Chair: **Chien-Fu Lin 林建甫** (CTBC Business School)

Climate Information Disclosure and Stock Market Volatility

Presenter: Guan-Yu Hou 侯冠宇 (National Kaohsiung University of Science and Technology)

Discussant: Chien-Lin Lu 盧建霖 (National Taipei University)

Firm's Rhetorical Patriotism and Green Innovation

Presenter: Chien-Lin Lu 盧建霖 (National Taipei University)

Discussant: Guan-Yu Hou 侯冠宇 (National Kaohsiung University of Science and Technology)

| | |
|--|---------------------------|
| SESSION 1-4 Cryptocurrency and FX markets | 10:05 – 11:05 Room 106 |
|--|---------------------------|

Session Chair: **Min-Teh Yu 俞明德** (National Tsing Hua University)

Cryptocurrency Momentum and Liquidity Volatility Premium

Presenter: Wei-Shau Chen 陳威劭 (Soochow University)

Discussant: Withz Aimable 艾馬坡 (Yuan Ze University)

Monetary Supply and Bitcoin Returns

Presenter: Po-Hsun Chuang 莊博勛 (Soochow University)

Discussant: Wei-Shau Chen 陳威劭 (Soochow University)

Disentangling Market and Uncertainty Effects in Crypto Valuation: A Portfolio-Based Analysis

Presenter: Withz Aimable 艾馬坡 (Yuan Ze University)

Discussant: Lei-Yi Ding 雷衣鼎 (National Chengchi University)

Academic Session I

Date: JUN 13rd, 2025 (Friday)

Venue: College of Management, CCU

| | |
|--|---|
| SESSION 1-5 Artificial intelligence and machine learning | 10:05 – 11:05 Room 119 |
|--|---|

Session Chair: **Hung-Wen Cheng 鄭宏文** (Soochow University)

Forecasting Credit Card Fraud via Machine Learning

Presenter: Xian-Ji Kuang 匡顯吉 (National Changhua University of Education)

Discussant: Jou-Ping Yu 尤柔蘋 (National Taiwan University)

Machine Learning in Momentum Strategies

Presenter: Jou-Ping Yu 尤柔蘋 (National Taiwan University)

Discussant: Yao-Tsung Chen 陳耀宗 (National Yang Ming Chiao Tung University)

Enhancing Performance of Investment Strategies Based on Text Categorization with NLP Techniques

Presenter: Yao-Tsung Chen 陳耀宗 (National Yang Ming Chiao Tung University)

Discussant: Xian-Ji Kuang 匡顯吉 (National Changhua University of Education)

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|---|---|
| SESSION 1-6 Insurance and risk management | 10:05 – 11:05 Room 120 |
|---|---|

Session Chair: **Yung-Tsung Lee 李永琮** (National Chiayi University)

Climate Risk and the Demand of Reinsurance in the Global Property-Liability Insurance Industry

Presenter: Yi-Hsun Lai 賴怡洵 (National Yunlin University of Science and Technology)

Discussant: Chia-Ling Ho 何佳玲 (Tamkang University)

Why Don't Insurers Hesitate? Investigating the Factors of Implementing Automated Claims Payment in Private Health Insurance

Presenter: Chia-Ling Ho 何佳玲 (Tamkang University)

Discussant: Yi-Hsun Lai 賴怡洵 (National Yunlin University of Science and Technology)

Academic Session II

Date: JUN 13rd, 2025 (Friday)

Venue: College of Management, CCU

SESSION 2-1
ESG and green finance

11:10 – 12:10
Room 102

Session Chair: **Lee-Young Cheng 鄭揚耀** (National Chung Cheng University)

Investigating the Role of Digital Transformation in ESG Performance: The Moderating Effect of Corporate Characteristics and the Impact of the External Environment

Presenter: Yi-Ching Huang 黃亦淨 (National Kaohsiung University of Science and Technology)

Discussant: Chia-Wei Yeh 葉家維 (National Chi Nan University)

The Influence of ESG Performance on Corporate Payout Policy: Evidence from Taiwan

Presenter: Chia-Wei Yeh 葉家維 (National Chi Nan University)

Discussant: Priyanka C. Bhatt (National Tsing Hua University)

Mapping ESG-Driven FinTech Innovation: A Patent Analytics Approach for Assessing Firm Innovations and Market Gaps

Presenter: Priyanka C. Bhatt (National Tsing Hua University)

Discussant: Yi-Ching Huang 黃亦淨 (National Kaohsiung University of Science and Technology)

SESSION 2-2
Corporate finance and governance

11:10 – 12:10
Room 103

Session Chair: **Hong-Ren Huang 黃泓人** (National Central University)

Deviation from target capital structure and corporate misconduct

Presenter: Ruey-Jenn Ho 何瑞鎮 (National Taichung University of Science and Technology)

Discussant: Hsuan-Ling Chang 張瑄凌 (Tamkang University)

Does Family Engagement Affect Operation Performance in the Banking Industry

Presenter: Tzu-Hao Hsu 徐子皓 (Mercuries Life Insurance)

Discussant: Ruey-Jenn Ho 何瑞鎮 (National Taichung University of Science and Technology)

Academic Session II

Date: JUN 13rd, 2025 (Friday)

Venue: College of Management, CCU

| | |
|---|---|
| SESSION 2-3 Quantitative trading and investment | 11:10 – 12:10 Room 105 |
|---|---|

Session Chair: **An-Sing Chen 陳安行** (National Chung Cheng University)

Head-and-shoulders patterns in the Taiwan stock market: A comprehensive analysis of predictive ability, profitability, and decision making factors

Presenter: An-Sing Chen 陳安行 (National Chung Cheng University)

Discussant: Jen-Wei Yang 楊仁維 (Soochow University)

Volatility Decay and Arbitrage in Leveraged ETFs: Evidence from the US and Japan

Presenter: Zong-Wei Yeh 葉宗瑋 (National Chengchi University)

Discussant: Hui-Ching Chuang 莊惠菁 (National Taipei University)

Classifying Hedge Fund Strategies with Large Language Models : Systematic vs. Discretionary Performance

Presenter: Hui-Ching Chuang 莊惠菁 (National Taipei University)

Discussant: Zong-Wei Yeh 葉宗瑋 (National Chengchi University)

| | |
|---|---|
| SESSION 2-4 Other topics related to financial markets | 11:10 – 12:10 Room 106 |
|---|---|

Session Chair: **Shu-Feng Wang 王樹鳳** (Ajou University)

Enhancing Data Analytical Skills: A Step-by-Step Spreadsheet Approach to Identifying and Understanding Top-Performing Stocks

Presenter: Te-Feng Chen (Hong Kong Polytechnic University)

Discussant: Chuin-Hong Chen 陳俊宏 (National Yunlin University of Science and Technology)

Order flow variability by investors' trade size choice and market returns

Presenter: Shu-Fang Yuan 袁淑芳 (Nanhua University)

Discussant: Te-Feng Chen (Hong Kong Polytechnic University)

The information content of share pledging effect on synchronous stock price movements

Presenter: Chuin-Hong Chen 陳俊宏 (National Yunlin University of Science and Technology)

Discussant: Shu-Fang Yuan 袁淑芳 (Nanhua University)

Academic Session II

Date: JUN 13rd, 2025 (Friday)

Venue: College of Management, CCU

SESSION 2-5

11:10 – 12:10

Corporate finance and governance

Room 119

Session Chair: **Her-Jiun Sheu 許和鈞** (Ming Chuan University)

Board gender diversity and firm performance: Empirical evidence from Taiwan

Presenter: Ching-Hua Yu 尤靜華 (Kainan University)

Discussant: Chiu-Ming Hsiao 蕭秋銘 (National Yunlin University of Science and Technology)

Board Gender Diversity and the Intraclass Pay Gap

Presenter: Yu-Lin Huang 黃幼琳 (National Taichung University of Science and Technology)

Discussant: Ching-Hua Yu 尤靜華 (Kainan University)

The Nexus between the Prices of Bitcoin, Carbon, and Electricity

Presenter: Chiu-Ming Hsiao 蕭秋銘 (National Yunlin University of Science and Technology)

Discussant: Yu-Lin Huang 黃幼琳 (National Taichung University of Science and Technology)

Academic Session III

Date: JUN 13rd, 2025 (Friday)

Venue: College of Management, CCU

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|--------------------------------------|
| SESSION 3-1 ESG and green finance |
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|---------------------------|
| 14:25 – 15:45 Room 102 |
|---------------------------|

Session Chair: Hsiu-Chuan Lee 李修全 (Ming Chuan University)

Institutional herding in positive and negative ESG performance and stock price crash risks

Presenter: Hao Fang 方豪 (Chung Yuan Christian University)

Discussant: Hsiu-Chuan Lee 李修全 (Ming Chuan University)

The Effect of ESG Performance and Sustainability Uncertainty on Mispricing: Empirical Analysis of Asian Developed Countries

Presenter: Lin-Chih Wu 吳蘭芝 (National Sun Yat-sen University)

Discussant: Hao Fang 方豪 (Chung Yuan Christian University)

The Impact of Sustainable Investment and R&D Innovation on Firm Value

Presenter: Min-Rui Choo 朱民芮 (Ming Chuan University)

Discussant: Lin-Chih Wu 吳蘭芝 (National Sun Yat-sen University)

Is the Commitment of Commercial Banks to Self-Regulation for Sustainable Development a CSR Practice or Greenwashing?

Presenter: Chien-Jung Li 李芊蓉 (National Taiwan University)

Discussant: Min-Rui Choo 朱民芮 (Ming Chuan University)

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| SESSION 3-2 Asset pricing: capital market efficiency and anomalies |
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|---------------------------|
| 14:25 – 15:45 Room 103 |
|---------------------------|

Session Chair: Larry Y. Tzeng 曾郁仁 (National Taiwan University)

Asset Pricing and A Tale of Night and Day: Evidence from Taiwan

Presenter: Hui-Wen Chang 張卉玟 (National Chengchi University)

Discussant: Chih-Hsiang Hsu 許智翔 (Ming Chuan University)

Do Net-buying Pressures on Institutional Investors Predict Stock Returns: Evidence from Taiwan Stock Market

Presenter: Kuang-Chieh Yen 顏廣杰 (Soochow University)

Discussant: Hui-Wen Chang 張卉玟 (National Chengchi University)

The High-volume Return Premium- The Evidence from the Taiwan Stock Market

Presenter: Mu-Shu Yun 雲慕書 (National Chung Cheng University)

Discussant: Kuang-Chieh Yen 顏廣杰 (Soochow University)

Return Seasonalities in the Taiwan Stock Market: The Role of Overnight and Daytime Returns

Presenter: Chih-Hsiang Hsu 許智翔 (Ming Chuan University)

Discussant: Mu-Shu Yun 雲慕書 (National Chung Cheng University)

Academic Session III

Date: JUN 13rd, 2025 (Friday)

Venue: College of Management, CCU

SESSION 3-3

14:25 – 15:45

Behavioral finance and risk management

Room 105

Session Chair: **Shih-Kuei Lin 林士貴** (National Chengchi University)

Employee Education and Stock Price Crash Risk

Presenter: Mei-Chen Lin 林美珍 (National Taipei University)

Discussant: Chien-Ping Chung 鍾建屏 (National Taipei University of Technology)

Option-Implied Probability Distortions and Stock Return Predictability

Presenter: Ting-Xuan Wang 王亭焜 (National Chengchi University)

Discussant: Meng-Lan Yueh 岳夢蘭 (National Chengchi University)

Frontier Culture and Firm Tax Avoidance

Presenter: Nhut Vu Truong Nguyen 葛同 (National Yang Ming Chiao Tung University)

Discussant: Mei-Chen Lin 林美珍 (National Taipei University)

Juicing Profits from Stock Dividends Using Random Forests

Presenter: Chien-Ping Chung 鍾建屏 (National Taipei University of Technology)

Discussant: Nhut Vu Truong Nguyen 葛同 (National Yang Ming Chiao Tung University)

SESSION 3-4

14:25 – 15:45

Artificial intelligence and machine learning

Room 106

Session Chair: **Wei-Hsien Li 李韋憲** (National Central University)

An Interpretable Model for U.S. Inflation Density Forecasting and Monitoring via Factor Augmentation and Machine Learning

Presenter: Shan-Ying Chu 朱珊瑩 (Chung Yuan Christian University)

Discussant: Chun-Tao Chuang 莊鈞洵 (National Chengchi University)

Enhancing the Financial Sustainability from Machine Learning

Presenter: Yu-Ying Tzeng 曾毓英 (National Chengchi University)

Discussant: Shan-Ying Chu 朱珊瑩 (Chung Yuan Christian University)

Machine Learning in Valuation, Market, Growth, and Risk Factors

Presenter: Yun-Xuan Hu 胡云瑄 (Academia Sinica)

Discussant: Yu-Ying Tzeng 曾毓英 (National Chengchi University)

The Application of Generative AI in Insurance Marketing: Enhancing Knowledge Management and Marketing Performance of Insurance Agents through the SECI Model

Presenter: Si-Yu Yang 楊思妤 (National Taipei University of Technology)

Discussant: Li-Han Chang 張琍韓 (Tamkang University)

Academic Session III

Date: JUN 13rd, 2025 (Friday)

Venue: College of Management, CCU

SESSION 3-5

14:25 – 15:45

Behavioral finance and risk management

Room 119

Session Chair: **Shuh-Chyi Doong 董樹琦** (National Chung Hsing University)

Climate Change Risk, Macroprudential Policy and Financial Stability

Presenter: Jui-Chuan Chang 張瑞娟 (National Chiayi University)

Discussant: Jui-Cheng Hung 洪瑞成 (Tamkang University)

Does Family Firm's Succession Affect Bank Loan Interest Rates? Evidence from Individual Bank Loan Contracts

Presenter: I-Hsuan Chang 張懿萱 (National Changhua University of Education)

Discussant: Jui-Chuan Chang 張瑞娟 (National Chiayi University)

Impact of Financial Reporting Quality on Behaviors of Institutional Investors in Taiwan

Presenter: Chi-Lu Peng 彭琪祿 (National Kaohsiung University of Science and Technology)

Discussant: Hung-Cheng Chang 張宏正 (National Yang Ming Chiao Tung University)

The impact of trading mechanism reforms on the disposition effect and overconfidence and: Evidence from Taiwan's stock market

Presenter: Jui-Cheng Hung 洪瑞成 (Tamkang University)

Discussant: Chi-Lu Peng 彭琪祿 (National Kaohsiung University of Science and Technology)

Academic Session III

Date: JUN 13rd, 2025 (Friday)

Venue: College of Management, CCU

SESSION 3-6
ESG and green finance

14:25 – 15:45
Room 120

Session Chair: **Yang-Che Wu** 吳仰哲 (Feng Chia University)

Beneficial to the Society but Harmful to Investors? Early Evidence on the Carbon Fee Policy in Taiwan

Presenter: Chun-Yen Wu 吳俊彥 (Chung-Hua Institution for Economic Research)

Discussant: Hsu-Jia Chen 陳毅嘉 (National Chengchi University)

Green Bonds, Financing Costs, and Corporate Value: An Empirical Analysis of Chinese Listed Companies

Presenter: Fu-Hsien Hsu 許馥顯 (National Chung Hsing University)

Discussant: Chun-Yen Wu 吳俊彥 (Chung-Hua Institution Economic Research)

The Crowding-Out Effect of ESG Investments in Capital-Intensive Industries: Evidence from Taiwan's Semiconductor and Electronics Firms During the COVID-19 Crisis

Presenter: Chia-Ching Lin 林佳慶 (Ling Tung University)

Discussant: Fu-Hsien Hsu 許馥顯 (National Chung Hsing University)

The Impact of ESG Information Disclosure on Merger and Acquisition: An Empirical Analysis

Presenter: Wei-Xiang Huang 黃偉翔 (Feng Chia University)

Discussant: Chia-Ching Lin 林佳慶 (Ling Tung University)

Academic Session IV

Date: JUN 13rd, 2025 (Friday)

Venue: College of Management, CCU

SESSION 4-1
ESG and green finance

16:00 – 17:20
Room 102

Session Chair: **Hung-Pin Lai 賴宏彬** (National Chung Cheng University)

The Relationship between Green Bond Issuance and Corporate Performance

Presenter: Wei-Ting Chen 陳曄婷 (Tamkang University)

Discussant: Masayo Shikimi (Nagasaki University)

Asian Suppliers' Innovation Response to Global Business Customers For Climate Risks

Presenter: Tse-Jen Lu 陸則壬 (National Chengchi University)

Discussant: Wei-Ting Chen 陳曄婷 (Tamkang University)

Materiality in Sustainability Reports and Information Quality: Evidence from Mandatory Industries in Taiwan

Presenter: Chou-Yen Wu 吳周燕 (Feng Chia University)

Discussant: Tse-Jen Lu 陸則壬 (National Chengchi University)

Climate Change Risks, ESG Disclosure, and Disinvestment: Evidence from Cross-shareholders

Presenter: Masayo Shikimi (Nagasaki University)

Discussant: Chou-Yen Wu 吳周燕 (Feng Chia University)

SESSION 4-2
Corporate finance and governance

16:00 – 17:20
Room 103

Session Chair: **Wei-Che Tsai 蔡維哲** (National Sun Yat-sen University)

Bootstrapping Analysis of ESG Indices in Commercial Banks

Presenter: Yang Li 李揚 (National Taiwan College of Performing Arts)

Discussant: Tzu-Chang Forrest Cheng 鄭子長 (National Sun Yat-sen University)

Stock Liquidity and Insider Trading Profitability

Presenter: Guan-Ying Huang 黃冠瑛 (National Chung Cheng University)

Discussant: Yang Li 李揚 (National Taiwan College of Performing Arts)

The effects of CEOs awards on corporate innovation: The role of investor attraction and talent attraction

Presenter: Wen-Ju Liao 廖文如 (Feng Chia University)

Discussant: Guan-Ying Huang 黃冠瑛 (National Chung Cheng University)

The Impact of the Cross-border E-commerce Pilot Zones on Digital Transformation: Evidence from China

Presenter: Tzu-Chang Forrest Cheng 鄭子長 (National Sun Yat-sen University)

Discussant: Wen-Ju Liao 廖文如 (Feng Chia University)

Academic Session IV

Date: JUN 13rd, 2025 (Friday)

Venue: College of Management, CCU

SESSION 4-3

16:00 – 17:20

Behavioral finance and risk management

Room 105

Session Chair: **Pei-Shih Weng** 翁培師 (National Sun Yat-sen University)

Outside CEOs and Borrowing Costs: Evidence from Taiwan

Presenter: Run-Chuan Qin 秦潤川 (National Yang Ming Chiao Tung University)

Discussant: Huong Thi Nguyen 阮氏香 (National Chung Cheng University)

The Lasting Impact of Inter-Racial Conflicts: Evidence from the Banking Markets

Presenter: Chih-Yung Lin 林智勇 (National Yang Ming Chiao Tung University)

Discussant: Pei-Shih Weng 翁培師 (National Sun Yat-sen University)

Stock Return Comovements and Investor Attention: Stocks or REITs

Presenter: Bai-Sian Chen 陳佰弦 (Chang Gung University)

Discussant: Run-Chuan Qin 秦潤川 (National Yang Ming Chiao Tung University)

Geopolitical risk and anti-herding behavior in the energy futures market

Presenter: Huong Thi Nguyen 阮氏香 (National Chung Cheng University)

Discussant: Bai-Sian Chen 陳佰弦 (Chang Gung University)

SESSION 4-4

16:00 – 17:20

Derivatives and risk management

Room 106

Session Chair: **Jui-Ching Huang** 黃瑞卿 (National Central University)

Order Spoofing, Price Impact, and Market Quality

Presenter: Pei-Fang Hsieh 謝佩芳 (National Tsing Hua University)

Discussant: Chi-Feng Tzeng 曾祺峰 (National Tsing Hua University)

The comparison of methods for extracting risk-neutral bankruptcy chance

Presenter: Chi-Feng Tzeng 曾祺峰 (National Tsing Hua University)

Discussant: Jie Luo 羅杰 (National Sun Yat-sen University)

The Impact of Large Retail Trading on Futures Prices

Presenter: Jie Luo 羅杰 (National Sun Yat-sen University)

Discussant: Pei-Lin Hsieh 謝沛霖 (National Chengchi University)

A Forward-Looking Index of Physical Market Variance

Presenter: Pei-Lin Hsieh 謝沛霖 (National Chengchi University)

Discussant: Pei-Fang Hsieh 謝佩芳 (National Tsing Hua University)

Academic Session IV

Date: JUN 13rd, 2025 (Friday)

Venue: College of Management, CCU

SESSION 4-5

16:00 – 17:20

Other topics related to financial markets

Room 119

Session Chair: **Jin-Huei Yeh 葉錦徽** (National Central University)

Assessing Impact of War on the Stock Markets of Participating Countries: A Comprehensive Event Analysis of War Declarations and Ceasefires

Presenter: Jin-Huei Yeh 葉錦徽 (National Central University)

Discussant: Wei-Hsien Li 李韋憲 (National Central University)

Decomposing Bid-Ask Spread - A Comparison between Aggregate ARMA and Poisson-HMM Models

Presenter: Ping-chen Tsai 蔡秉真 (National Sun Yat-sen University)

Discussant: Calvin J. Chiou 邱健嘉 (National Chengchi Univesity)

Analytic Formulae for Valuing Guaranteed Minimum Withdrawal Benefits under Stochastic Interest Rates

Presenter: I-Chien Liu 劉議謙 (National Taichung University of Science and Technology)

Discussant: Ping-chen Tsai 蔡秉真 (National Sun Yat-sen University)

Common Ownership and Board Interlocks: Industry Boundary Matters

Presenter: Calvin J. Chiou 邱健嘉 (National Chengchi Univesity)

Discussant: I-Chien Liu 劉議謙 (National Taichung University of Science and Technology)

SESSION 4-6

16:00 – 17:20

Other topics related to financial markets

Room 120

Session Chair: **Yu-Sheng Lai 賴兩聖** (National Chi Nan University)

Co-movement in Cryptocurrency and Stock Market: Spillover Effects of Return and Volatility around COVID-19 Pandemic

Presenter: Chia-Hung Chen 陳嘉宏 (National Yunlin University of Science and Technology)

Discussant: Yun-Chieh Lu 呂昀潔 (National Chengchi Univesity)

Exploring the Impact of Accounting Regulation Changes on Incentive Compensation Structure Using Structural Credit Risk Models

Presenter: Yi-Chou Liu 劉易洲 (National Yang Ming Chiao Tung University)

Discussant: Chia-Hung Chen 陳嘉宏 (National Yunlin University of Science and Technology)

Banks' Adaptation of Green Lending and Risk-Taking Under Mandatory TCFD Disclosure: Evidence from Taiwan

Presenter: Yun-Chieh Lu 呂昀潔 (National Chengchi Univesity)

Discussant: Yi-Chou Liu 劉易洲 (National Yang Ming Chiao Tung University)

PACIFIC-BASIN FINANCE JOURNAL

Conference Special Issue



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Manuscript Submission Deadline: 30 September 2025

Editorial Acceptance Deadline: 30 June 2026

2025 台灣財務工程學會年會暨國際學術研討會 《人文及社會科學集刊》「行為財務學特刊」 徵稿啟事

■特刊編輯

蔡明宏（中央研究院人文社會科學研究中心副研究員）

翁培師（中山大學財務管理學系教授）

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3. 在本屆研討會論文發表之後且有意願投稿者，請於 2025 年 9 月 30 日之前於《人文及社會科學集刊》線上投稿系統「行為財務學特刊」專區投稿。我們將儘速安排以本刊一貫標準進行審查。
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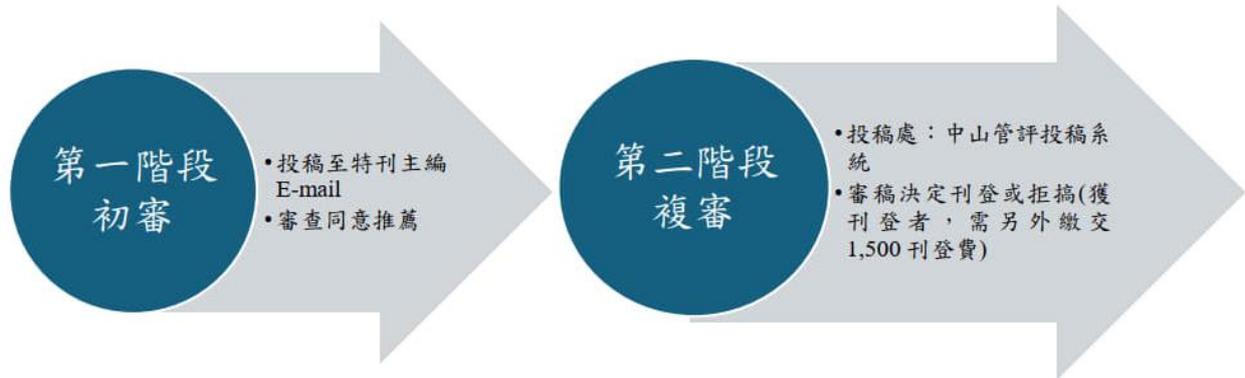
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2025 FeAT 年會暨國際研討會「中山管理評論」特約投稿說明

2025 FeAT 年會暨國際研討會與中山管理評論 (TSSCI) 合作，研討會結束後投稿流程說明：



1. 第一階段 (學會初審)：在本屆研討會論文發表之後且有意願投稿至中山管理評論者，請在 2025 年 8 月 31 號之前將文章投稿至特刊主編林子綾教授(電子信箱：tzuling@ccu.edu.tw)。第一階段編輯委員會派送審稿委員，決定是否「同意推薦」。

➤ **第一階段編輯委員會：**

- 林子綾教授 (國立中正大學財務金融學系)
- 盧建霖副教授 (國立台北大學企業管理學系)
- 蔡維哲特聘教授 (國立中山大學財務管理學系兼中山管理評論財務會計領域主編)

➤ **截止日期：**2025 年 8 月 31 日

➤ **投稿電子信箱：**tzuling@ccu.edu.tw

➤ **投稿格式：**論文需含封面頁，註明標題及摘要、作者姓名、服務單位、職稱、聯絡地址、聯絡電話、電子郵箱。來稿請以 MS Word 或 PDF 檔。

2. 第二階段 (中山管評複審)：學會會通知前階段獲「同意推薦」者，可針對審查意見修改之後，於 2025 年 11 月 30 日之前投稿至中山管理評論投稿系統 (<https://mgtr.cm.nsysu.edu.tw/>)，其後由中山管理評論另外進行僅有一輪的審稿，審稿結束即會決定是否「推薦刊登」。接獲「推薦刊登」者，需再另外繳交 1,500 元的刊登費。

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