國立中正大學103學年度學士班三年級轉學生招生考試試題

學系別：財務金融學系　　科目：微積分

第 1 節　　

Please show all your work.

1. (20 pts.) Evaluate each of the followings and simplify your answer.
   (a) $\frac{d}{dx}x^{\ln x}$ (b) $\int_{-\infty}^{\infty}\frac{x}{x^2+1}dx$ (c) $\frac{d}{dx}\int_{1}^{x^3}\frac{t^2+1}{t+2}dt$ (d) $\int_{-1}^{1}\left|x^2-x\right|dx$

2. (20 pts) Sketch the graph of $f(x)=\frac{x^3+3x^2}{x^2+2x-3}$, and determine **(i)** domain and range, **(ii)** intervals increasing or decreasing, **(iii)** relative extrema, **(iv)** intervals of concave up or concave down, **(v)** inflection points, **(vi)** intercepts and **(vii)** asymptotes.

3. (10 pts.) Frank owns a stamp collection that is currently worth \$1,200 and whose value increases linearly at the rate of \$200 per year. If the prevailing interest rate remains constant at 6% compounded continuously, when will it be most advantageous for him to sell the collection and invest the proceeds?

4. (15 pts) A manufacturer of machinery part determines that q thousand units of a particular piece will be sold when the price is $p=S(q)=5+q$ dollars per unit. The total cost of producing q thousand units is $C(q)=q^3-3q^2+2q+3000$ thousand dollars.
   (a) For what value of q is profit maximized?
   (b) Find the producers' surplus for the level of production corresponding to maximum profit.
   (c) Let the demand function for the machinery part be $D(q)=\sqrt{245-2q}$, find the consumers' surplus at equilibrium.

5. (15 pts.) Suppose the utility derived by a consumer from x units of one commodity and y units of a second commodity is given by the utility function $U(x,y)=x^3y^2$.
   (a) The consumer currently owns 5 units of the first commodity and 4 units of the second. Use calculus to estimate how many units of the second commodity the consumer could substitute for 1 unit of the first commodity without affecting total utility.
   (b) Suppose the first commodity costs \$50 per unit and the second commodity cost \$20 per units, if you have \$1,000, how should you allocate the money to generate maximum utility?
   (c) If you have extra \$100, use the Lagrange multiplier to estimate how this change will affect the maximum utility.

6. (10 pts.) Find the critical points of $f(x,y)=x^4+y^4-2x^2-2y^2$, and classify each point as a relative maximum, a relative minimum, or a saddle point.

7. (10 pts.) Use double integration to find the average value of $f(x,y)=18xy^2$ over the triangle with vertices (0, 0), (0, 2), and (3, 2).