Please write down your calculations in detail for Problem $2 \sim 9$.

1. (10 pts) True(○) or False(X).
   (a) (_____) A polynomial function of degree 3 has at most one inflection point.
   (b) (_____) Let $f(x) = -x^3 + 2x^2 - 28x - 1250$. Then $f(x)$ has a negative real root.
   (c) (_____) Let $f(x)$ be a continuous function and $g(x)$ be a differentiable function. Then $\int_a^b f(g(x))g'(x)dx = \int_a^b f(u)du$.
   (d) (_____) If $p < 0$, then $\int_1^\infty e^{px}dx$ is convergent.
   (e) (_____) If $f(x, y)$ has a relative maximum at the point (a, b), then $f_x(a, b) = f_y(a, b) = 0$.

2. (20 pts) Evaluate each of the followings:
   $(a)\ \frac{d}{dx}[x\ln(2x^2+5)]$; $(b)\ \frac{d}{dx}\int_{x^2}^1 \sqrt{t^2+t+1}dt$; $(c)\ \int_1^e \ln x dx$; $(d)\ \int_0^\infty xe^{-x^2}dx$.

3. (12 pts) Consider the function $f(x) = \frac{2x^2}{x^2-1}$.
   (a) Find the domain of f.
   (b) Find the intervals where f is increasing and the intervals where f is decreasing.
   (c) Find all extrema, if any, of f.
   (d) Find the intervals where f is concave up and the intervals where f is concave down.
   (e) Find all the asymptotes.
   (f) Sketch the graph of f.

4. (10 pts) A video store has estimated that the rental price p (in dollars) of DVDs is related to the quantity x (in thousands) rented/day by the demand equation

$$x = \frac{2}{3}\sqrt{36 - p^2} \quad (0 \le p \le 6).$$

   (a) Compute the elasticity of demand $E(p) = \frac{px'(p)}{x(p)}$. (6%)
   (b) The rental price is currently 2 dollars/disc. If the rental price is increased, will the revenue increase or decrease? Why? (4%)

5. (8 pts) Assume that a function f satisfies $f(0) = 4$ and the slope of the tangent line at each point $(x, f(x))$ is xe^x. Find the function $f(x)$.

6. (10 pts) Suppose that we are given 5 data points:

$$(x_1, y_1) = (1, 1),\ (x_2, y_2) = (2, 3),\ (x_3, y_3) = (3, 4),\ (x_4, y_4) = (4, 3),\ (x_5, y_5) = (5, 6).$$

   Let $y = mx + b$ be the least-squares line of the 5 data points and F be a function of m and b defined by $F(m, b) = (y_1 - mx_1 - b)^2 + \cdots + (y_5 - mx_5 - b)^2$.

   (a) Evaluate and simplify $\frac{\partial F}{\partial m}(m, b)$ and $\frac{\partial F}{\partial b}(m, b)$. (6%)

   (b) Find the minimum of the function F and the value of m and b when F attains its minimum. (4%)

7. (10 pts) Find the maximum and minimum values of the function $f(x, y) = e^{xy}$ subject to the constraint $x^2 + y^2 = 1$.

8. (10 pts) Evaluate the double integral $\int\int_R e^{-x^2} dA$, where R is the triangular region with vertices $(0, 0)$, $(1, 0)$, $(1, 1)$.

9. (10 pts)

   (a) Find the range of x such that the sequence $\left\{\left(\frac{2x}{x+2}\right)^n\right\}_{n=1}^{\infty}$ converges.

   (b) If $\sum_{n=1}^{\infty}\left(\frac{2x}{x+2}\right)^n = -x$, find the value of x.