

Please show all your work.

1. (28 pts.) Evaluate each of the followings and simplify your answer.

$$(a) \frac{d}{dx} x^x \quad (b) \int_1^4 (\sqrt{x} + 1) \ln x \, dx \quad (c) \int_1^{\infty} \frac{1}{(1-x)^2} \quad (d) \int_0^1 \int_y^{\sqrt{y}} e^{x^2} \, dx dy$$

2. (10 pts.) Find the critical points of $f(x, y) = \frac{x}{x^2 + y^2 + 1}$, and classify each point as a relative maximum, a relative minimum, or a saddle point.

3. (12 pts) Graph the function $f(x) = (x+1)e^{-\frac{x}{2}}$, and (i) determine the domain and range, (ii) intervals increasing or decreasing, (iii) relative extrema, (iv) intervals of concave up or concave down, (v) inflection points, (vi) intercepts and (vii) asymptotes.

4. (10 pts) If x thousand dollars is spent on labor and y thousand dollars is spent on equipment, the output at a certain factory is modeled by $Q(x, y) = 50x^{\frac{3}{2}}y^{\frac{1}{2}}$. Assume \$8000 is available, how should money be allocated to generate the possible maximum output? And, use the Lagrange multiplier to estimate the change in the maximum output if the money available is increased by \$500.

5. (10 pts) Find the equation of the tangent line to the curve $y = f(x) = \int_{x^3}^1 \frac{dt}{\sqrt{1+3t^2}}$ at the point where $x = 1$.

6. (15 pts) At age 25, Ann starts her job with salary which is starting \$600,000 and increasing exponentially with rate 1% per year. Assume that she retires at age 65.
- Find the average salary per year?
 - Assume the prevailing annual interest rate remains fixed at 2% compounded Continuously. What is the present value of her job?
 - Ann deposits 20% of her salary into a bond fund that pays 4% annual interest compounded continuously. Assume that her deposits are made as a continuously income flow. How much money will be in the account when she retires?

7. (15 pts) When a particular commodity is priced at p dollars per unit, consumer demand q units, where p and q are related by the equation $0.01p^2 + q = 256$. The price in terms of the unit cost c is expressed as $p(c) = 2c - 10$.

- Find the elasticity of demand, $E = -\frac{p}{q} \frac{dq}{dp}$, of the commodity for unit price of \$100. And, give an interpretation of you answer.
- What price results maximum revenue?
- If the cost is increasing 1% at the price \$50, use the differential to approximate the changes in total profit.