

Please show all your work.

1. (20 pts.) Evaluate each of the followings and simplify your answer.

(a) $\frac{d}{dx} x^{\ln x}$ (b) $\int_{-\infty}^{\infty} \frac{x}{x^2 + 1} dx$ (c) $\frac{d}{dx} \int_1^{x^3} \frac{t^2 + 1}{t + 2} dt$ (d) $\int_{-1}^1 |x^2 - x| dx$

2. (20 pts) Sketch the graph of $f(x) = \frac{x^3 + 3x^2}{x^2 + 2x - 3}$, and determine (i) domain and range, (ii) intervals increasing or decreasing, (iii) relative extrema, (iv) intervals of concave up or concave down, (v) inflection points, (vi) intercepts and (vii) asymptotes.

3. (10 pts.) Frank owns a stamp collection that is currently worth \$1,200 and whose value increases linearly at the rate of \$200 per year. If the prevailing interest rate remains constant at 6% compounded continuously, when will it be most advantageous for him to sell the collection and invest the proceeds?

4. (15 pts) A manufacturer of machinery part determines that q thousand units of a particular piece will be sold when the price is $p = S(q) = 5 + q$ dollars per unit. The total cost of producing q thousand units is $C(q) = q^3 - 3q^2 + 2q + 3000$ thousand dollars.

(a) For what value of q is profit maximized?

(b) Find the producers' surplus for the level of production corresponding to maximum profit.

(c) Let the demand function for the machinery part be $D(q) = \sqrt{245 - 2q}$, find the consumers' surplus at equilibrium.

5. (15 pts.) Suppose the utility derived by a consumer from x units of one commodity and y units of a second commodity is given by the utility function $U(x, y) = x^3 y^2$.

(a) The consumer currently owns 5 units of the first commodity and 4 units of the second. Use calculus to estimate how many units of the second commodity the consumer could substitute for 1 unit of the first commodity without affecting total utility.

(b) Suppose the first commodity costs \$50 per unit and the second commodity cost \$20 per units, if you have \$1,000, how should you allocate the money to generate maximum utility?

(c) If you have extra \$100, use the Lagrange multiplier to estimate how this change will affect the maximum utility.

6. (10 pts.) Find the critical points of $f(x, y) = x^4 + y^4 - 2x^2 - 2y^2$, and classify each point as a relative maximum, a relative minimum, or a saddle point.

7. (10 pts.) Use double integration to find the average value of $f(x, y) = 18xy^2$ over the triangle with vertices $(0, 0)$, $(0, 2)$, and $(3, 2)$.