

PART I. Multiple Choice Questions
INSTRUCTIONS: You do

- Which of the following is true?
 - If event A and B are mutually exclusive, then A and B are not independent.
 - If event A and B are not independent, then the covariance of A and B must be independent.
 - The relationship between A and B where a and b are correlation coefficients must equal to 1.
- Firm ABC is concerned about its stock price. It estimates that the probability of ABC going below its book value of \$50 is 0.7. The probability of ABC going below 50, given that XYZ is the probability that XYZ does not drop below 50, is 0.6.
 - 0.8667
 - 0.6
- The probability density function of X is given by $f(x) = 6x(1-x), 0 \leq x \leq 1$.
 - 1/6
 - 1/3
- Random variable X denotes the number of customers arriving at a bank. We also know that $E(X) = 100$. Use Chebyshev's Inequality to find the upper bound on the probability that the number of customers is outside the range [75, 125], which is
 - 2/125
 - 4/125
- The joint distribution of X and Y is given by $f(x, y) = 2x, 0 \leq x \leq 1, 0 \leq y \leq 1$.
 - 1/6
 - 1/3
- Suppose we cut a horizontal rod of length 5 feet into two pieces randomly. Let X be the random variable representing the length of the piece on the left hand side. Correspondingly, 5-X is the length of the piece on the right hand side. Find the expectation of the cross-sectional area of the rod, which is: (feet²)
 - 5/2
 - 25/2
- There are 400 identical machines in a factory. The probability of failure of each machine is 0.02. Use Poisson approximation to estimate the probability that more than one machine are fail.

3 points each.

Do not show calculation steps (無需列出計算過程).

If event A and B are mutually exclusive, then A and B are not independent. If the covariance of A and B equals to zero, then X and Y

The relationship between X and Y is that $Y = a + bX$, where a and b are correlation coefficient of X and Y

Firm ABC is concerned about its stock price. It estimates that the probability of ABC going below its book value of \$50 is 0.7. The probability of ABC going below 50, given that XYZ is the probability that XYZ does not drop below 50, is 0.6. What is the probability that ABC stock

The probability density function of X is given by

Random variable X denotes the number of customers arriving at a bank. We also know that $E(X) = 100$. Use Chebyshev's Inequality to find the upper bound on the probability that the number of customers is outside the range

[75, 125], which is

The joint distribution of X and Y is given by $f(x, y) = 2x, 0 \leq x \leq 1, 0 \leq y \leq 1$. The covariance of X and Y is

Suppose we cut a horizontal rod of length 5 feet into two pieces randomly. Let X be the random variable representing the length of the piece on left hand side. Correspondingly, 5-X is the length of the piece on the right hand side. Find the expectation of the cross-sectional area of the rod, which is: (feet²)

There are 400 identical machines in a factory. The probability of failure of each machine is 0.02. Use Poisson approximation to estimate the probability that more than one machine are fail.

8. Suppose X follows an exponential distribution ($f(x) = \frac{1}{\mu} e^{-x/\mu}$, $0 < x < \infty$), and

$P(x \leq 1) = P(x > 1)$, then $\text{Var}(x)$ is

- (a). $2\ln(2)$ (b). $\ln(2)$ (c). e^{-4} (d). e^{-2}

9. Let $X_1, X_2, \dots, X_n, X_{n+1}, \dots, X_{m+n}$ be random samples from a population with

distribution $\sim N(0, \sigma^2)$. Let $Y = \frac{\sqrt{m} \sum_{i=1}^n X_i}{\sqrt{n} \sqrt{\sum_{i=n+1}^{n+m} X_i^2}}$. Then Y has;

- (a). a t distribution with degree of freedom $df=n$.
 (b). a t distribution with degree of freedom $df=n-1$.
 (c). a t distribution with degree of freedom $df=m$.
 (d). a t distribution with degree of freedom $df=m-1$.

10. Let $X_1, X_2, \dots, X_n, X_{n+1}, \dots, X_{m+n}$ be random samples from a population with

distribution $\sim N(0, \sigma^2)$. Let $Y = \frac{m \sum_{i=1}^n X_i^2}{n \sum_{i=n+1}^{n+m} X_i^2}$. Then; Y has

- (a). a F distribution with degree of freedom $df_1 = n-1$, $df_2 = m-1$
 (b). a F distribution with degree of freedom $df_1 = n$, $df_2 = m$
 (c). a F distribution with degree of freedom $df_1 = m-1$, $df_2 = n-1$
 (d). a F distribution with degree of freedom $df_1 = m$, $df_2 = n$

11. Survey the income from 25 persons and find their sample mean is 35 (thousand).

Suppose its distribution is unknown but the population standard deviation of a person's income is known to be 6. Use Chebyshev inequality to construct its 96% confidence interval for average income, which is

- (a). [32, 38] (b). [30, 40] (c). [29, 41] (d). [28, 42]

12. The *p.d.f.* of random variable X is $f(x) = \frac{1}{\mu} e^{-x/\mu}$, $0 < x < \infty$. To test the

hypothesis $H_0: \mu = 10$ vs. $H_a: \mu = 20$. Let $X > 15$ be the rejection region. Then, the probability of Type I error (α) is

- (a). $1 - e^{-3/2}$ (b). $e^{-3/2}$ (c). $1 - e^{-3/4}$ (d). $e^{-3/4}$

13. Following question 12, the testing power ($1 - \beta$) is.

- (a). $1 - e^{-3/2}$ (b). $e^{-3/2}$ (c). $1 - e^{-3/4}$ (d). $e^{-3/4}$

14. The *p.d.f.* of random variable X is $f(x) = (\alpha + 1)x^\alpha$, $0 < x < 1$. Then, the method of moment estimator (MOM) for parameter α is (Hint: let $E(X) = \bar{X}$):

- (a). $\frac{1-2\bar{X}}{\bar{X}-1}$ (b). $\frac{2\bar{X}-1}{\bar{X}-1}$ (c). $\frac{\bar{X}-1}{2\bar{X}-1}$ (d). $\frac{1-\bar{X}}{2\bar{X}-1}$

15. Which of the following statements about the analysis of variance (ANOVA) is incorrect?
- (a). Under the assumption of common variance, mean square for error is an unbiased estimator for the variance
 - (b). Under the assumption of common variance, mean square for treatment may be used as an unbiased estimator for the variance
 - (c). Using t -test to test the equivalence of paired treatment mean, given the same data, may increase the actual type I error.
 - (d). To test the equivalence of treatment mean, using randomized block design is always more powerful than completely randomized design.
16. Consider a regression with two independent variables. If we find the correlation coefficient between the two independent variables approaches to one, then
- (a). the variance inflation factor will approach to zero
 - (b). the variance inflation factor will approach to infinity
 - (c). the standard deviation of the slopes will approach zero
 - (d). the standard deviation of the slopes will approach infinity
17. Consider a regression with sample size $n=40$, you are told that $\sum y_i = 160$, $\bar{y} = 80$ and the Least Square estimate of β_0 (intercept) is 2, then β_1 (slope) = ?
- (a). 1/2
 - (b). 1
 - (c). 2
 - (d). 4
18. Which of the following is false? The Least Square estimator for regression coefficient β_1 is unbiased if
- (a). is a linear combination of observation y
 - (b). is consistent only if independent variables and error term are independent
 - (c). can not be biased if the independent variables are linearly dependent
 - (d). can not be biased if Gauss Markov assumptions do not hold
19. When fitting a regression model $Y = \alpha + \beta X + \varepsilon$, you find that the error variance is proportional to $X^{-1/2}$. Which of the following models can be used to correct the above for heteroskedasticity?
- (a). $YX^{1/4} = \alpha + \beta X^{5/4} + \varepsilon^*$
 - (b). $YX^{1/4} = \alpha + \beta X^{5/4} + \varepsilon^*$
 - (c). $YX^{1/2} = \alpha + \beta X^{3/2} + \varepsilon^*$
 - (d). $YX^{-1/4} = \alpha + \beta X^{3/4} + \varepsilon^*$
 - (e). $YX^{-1/2} = \alpha + \beta X^{1/2} + \varepsilon^*$
20. A financial manager uses firm's operating profit income from previous year as the dependent variable and firm's leverage in the current year as the independent variable in a simple regression. Using 20 years of data, the manager estimates the model slope coefficient with the least square estimator $\hat{\beta}$, but does not take into account that the error term in the model exhibit to be first order autoregressive with autocorrelation coefficient $\rho > 0$. Which of the following statements is false?

- (a). The R^2 probably gives an overly optimistic picture of the success of the regression.
- (b). The estimator of the standard deviation of $\hat{\beta}$ is biased downward.
- (c). The estimator $\hat{\beta}$ is inconsistent.
- (d). The estimator $\hat{\beta}$ is inefficient.

PART II.

INSTRUCTIONS : There are three questions worth a total of 40 points in Part II. It is necessary to show the detail solution processes on your answer sheet. (必須詳列其計算過程，只寫答案不予計分)

1. Suppose that financial crisis occurs in accordance with the assumption of the Poisson probability distribution at a rate of 1 times per five years.
 - (1). Find the probability that at least twice financial crises during next decay.
 - (2). Find the probability distribution of the time between now and the next earthquake. (16%)

2. Let P be the probability of getting a head when a given coin is tossed. In order to test $H_0: p = 0.5$ vs. $H_1: p \neq 0.5$, a trial, tossing the coin 5 times, is conducted, and let X be the number of head obtained. If a test rejects H_0 when $X = 0$ and $X = 5$, answer the following questions.
 - (1) Find the probability of type I error
 - (2) If $p = 0.6$, find the probability of type II error.
 - (3) If $p = 0.6$, find the power of test.
 - (4) If the trial obtains 4 head, find the p-value of this test. (16%)

3. An economist wishes to study the monthly trend in the Dow Jones Industrial Average (DJIA). Data collected over the past 40 months were used to fit the model, where y_t =monthly close _{t} of the DJIA and t =month (1,2,...,40). The regression results appear below:
$$\hat{y}_t = 28 + 0.15y_{t-1} \quad R^2 = 0.66 \quad MSE = 144 \quad t_{int} = 1.72 \quad t_{y_{t-1}} = 3.76 \quad DW = 1.32$$
Conduct a test to determine if the residuals are positively correlated (you need to state hypotheses, level of significant, decision rules,.....etc) (8%)

附表 3. Table of normal distribution

Z Table

Entries in the body of the table represents areas under the curve between $-\infty$ and z

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

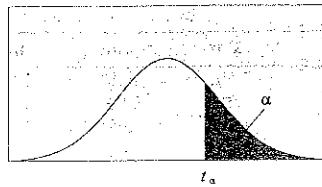
附表 1. Durbin-Watson critical values

X variables, excluding the intercept

Observations N	Prob.	1		2		3		4		5	
		D-L	D-U	D-L	D-U	D-L	D-U	D-L	D-U	D-L	D-U
15	0.05	1.08	1.36	0.95	1.54	0.82	1.75	0.69	1.97	0.56	2.21
	0.01	0.81	1.07	0.7	1.25	0.59	1.46	0.49	1.70	0.39	1.96
20	0.05	1.20	1.41	1.10	1.54	1.00	1.68	0.90	1.83	0.79	1.99
	0.01	0.95	1.15	0.86	1.27	0.77	1.41	0.68	1.57	0.60	1.74
25	0.05	1.29	1.45	1.21	1.55	1.12	1.66	1.04	1.77	0.95	1.89
	0.01	1.05	1.21	0.98	1.30	0.90	1.41	0.83	1.52	0.75	1.65
30	0.05	1.35	1.49	1.28	1.57	1.21	1.65	1.14	1.74	1.07	1.83
	0.01	1.13	1.26	1.07	1.34	1.01	1.42	0.94	1.51	0.88	1.61
40	0.05	1.44	1.54	1.39	1.60	1.34	1.66	1.39	1.72	1.23	1.79
	0.01	1.25	1.34	1.20	1.40	1.15	1.46	1.10	1.52	1.05	1.58
50	0.05	1.50	1.59	1.46	1.63	1.42	1.67	1.38	1.72	1.34	1.77
	0.01	1.32	1.40	1.28	1.45	1.24	1.49	1.20	1.54	1.16	1.59
60	0.05	1.55	1.62	1.51	1.65	1.48	1.69	1.44	1.73	1.41	1.77
	0.01	1.38	1.45	1.35	1.48	1.32	1.52	1.28	1.56	1.25	1.60
80	0.05	1.61	1.66	1.59	1.69	1.56	1.72	1.53	1.74	1.51	1.77
	0.01	1.47	1.52	1.44	1.54	1.42	1.57	1.39	1.60	1.36	1.62
100	0.05	1.65	1.69	1.63	1.72	1.61	1.74	1.59	1.76	1.57	1.78
	0.01	1.52	1.56	1.50	1.58	1.48	1.60	1.46	1.63	1.44	1.65

附表 2. t-Table t 分配臨界值表

$$P(t > t_{\alpha}) = \alpha$$



d.f.	t.100	t.050	t.025	t.010	t.005	d.f.
1	3.078	6.314	12.706	31.821	63.656	1
2	1.886	2.920	4.303	6.965	9.925	2
3	1.638	2.353	3.182	4.541	5.841	3
4	1.533	2.132	2.776	3.747	4.604	4
5	1.476	2.015	2.571	3.365	4.032	5
6	1.440	1.943	2.447	3.143	3.707	6
7	1.415	1.895	2.365	2.998	3.499	7
8	1.397	1.860	2.306	2.896	3.355	8
9	1.383	1.833	2.262	2.821	3.250	9
10	1.372	1.812	2.228	2.764	3.169	10
11	1.363	1.796	2.201	2.718	3.106	11
12	1.356	1.782	2.179	2.681	3.055	12
13	1.350	1.771	2.160	2.650	3.012	13
14	1.345	1.761	2.145	2.624	2.977	14
15	1.341	1.753	2.131	2.602	2.947	15
16	1.337	1.746	2.120	2.583	2.921	16
17	1.333	1.740	2.110	2.567	2.898	17
18	1.330	1.734	2.101	2.552	2.878	18
19	1.328	1.729	2.093	2.539	2.861	19
20	1.325	1.725	2.086	2.528	2.845	20
21	1.323	1.721	2.080	2.518	2.831	21
22	1.321	1.717	2.074	2.508	2.819	22
23	1.319	1.714	2.069	2.500	2.807	23
24	1.318	1.711	2.064	2.492	2.797	24
25	1.316	1.708	2.060	2.485	2.787	25
26	1.315	1.706	2.056	2.479	2.779	26
27	1.314	1.703	2.052	2.473	2.771	27
28	1.313	1.701	2.048	2.467	2.763	28
29	1.311	1.699	2.045	2.462	2.756	29
30	1.310	1.697	2.042	2.457	2.750	30
31	1.310	1.696	2.040	2.453	2.744	31
32	1.309	1.694	2.037	2.449	2.739	32
33	1.308	1.692	2.035	2.445	2.733	33
34	1.307	1.691	2.032	2.441	2.728	34
35	1.306	1.690	2.030	2.438	2.724	35
36	1.306	1.688	2.028	2.435	2.720	36
37	1.305	1.687	2.026	2.431	2.715	37
38	1.304	1.686	2.024	2.429	2.712	38
39	1.304	1.685	2.023	2.426	2.708	39
40	1.303	1.684	2.021	2.423	2.705	40