

臺灣綜合大學系統 106 學年度學士班轉學生聯合招生考試試題

科目名稱	微積分 B	類組代碼	
		科目碼	E0012

※本項考試依簡章規定各考科均「不可以」使用計算機

本科試題共計 1 頁

*** Show All Your Work. No Electronic Devices Allowed. ***

1.(10%) Find the tangent line of the curve $3x^2 - 2xy^2 + y^4 = 9$ at the point $(2, 1)$.

2.(10%) Find the intervals on which $f(x) = \frac{\ln x}{\sqrt{x}}$ ($x > 0$) is increasing or decreasing.

3.(10%) Let $F(x) = \int_{\frac{2}{x}}^{x+1} \sqrt{2t^2 + t + 3} dt$ ($x > 0$). Evaluate $F'(2)$ and $(F^{-1})'(0)$.

4.(10%) Evaluate the limit. $\lim_{x \rightarrow 0^+} \frac{\sin(\sqrt{x}) - \sqrt{x}}{x\sqrt{x}}$

5.(10%) Evaluate the definite integral. $\int_0^{\frac{\ln 2}{2}} e^t \sqrt{4 - e^{2t}} dt$

6.(10%) Evaluate the improper integral. $\int_0^1 \frac{\ln x}{\sqrt{x}} dx$

7.(10%) Use $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ to express the value $\int_0^{\frac{1}{2}} \frac{1}{2+x^3} dx$ as an infinite series.

8.(10%) Find the maximum rate of change of $f(x, y) = xe^{\frac{y}{x}}$ at point $(1, 3)$.

9.(10%) Use the method of Lagrange multiplier to maximize the profit of a product

$$\frac{150x}{x+5} + \frac{160y}{y+3} \quad (x: \text{development}, y: \text{promotion}) \text{ subject to the budget } x + y = 100.$$

10.(10%) Evaluate the double integral. $\int_0^1 \int_0^x \frac{y^3}{(x+y^2)^2} dy dx$