國立中正大學
111 學年度碩士班招生考試
試題
[第2節]

科目名稱	統計學
系所組別	財務金融學系

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始 40 分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

Part I. blank filling question:

Total 50% and 5% for each blank

1. Suppose probabilities of the two events C_1 and C_2 are $\Pr(C_1) = \Pr(C_2) = 4/7$. The probability of $\Pr(C_1 \cap C_2)$ should be at least __(1)__.

2. Let X_1 and X_2 have the joint probability density function: $f(x_1, x_2) = 2$, for $0 < x_1 < x_2 < 1$; and zero elsewhere. Then $\Pr(0 < X_1 < 1/2) =$ __(2)__ and $\Pr(0 < X_1 < 1/2 | X_2 = 3/4) =$ __(3)__.

3. Assume that $X_i \sim i.i.d.N(0,1)$, $i = 1, 2$, and $Y_j \sim i.i.d.N(1,1)$, $j = 1, 2, 3$, where X_i and Y_j are mutually independent for all i and j. Please answer the following questions: (**Note**: You should precisely write down the parameter(s) of each distribution in your answers, such as the degree of freedom or the mean and variance parameters in a normal distribution.)

   (a) Let $Q_1 = \frac{1}{2}\sum_{i=1}^{2} X_i + \frac{1}{3}\sum_{j=1}^{3} Y_i$, then the distribution of Q_1 is __(4)__.

   (b) Let $Q_2 = X_1^2 + X_2^2 + (Y_3 - 1)^2$, then the distribution of Q_2 is __(5)__.

   (c) Let $Q_3 = \frac{Y_3 - 1}{\sqrt{(X_1^2 + X_2^2)/2}}$, then Q_3 follows the __(6)__ distribution.

   (d) Let $Q_4 = \frac{2X_1^2}{X_2^2 + (Y_3 - 1)^2}$, then the distribution of Q_4 is __(7)__.

4. Suppose that random variable Y has a probability density function given by

$$f(y) = \begin{cases} k(1-y)y^2 & 0 \le y \le 1, \\ 0 & \text{elsewhere.} \end{cases}$$

then $k=$ __(8)__. The variance of Y is __(9)__. The expected value of Y^{-1} is __(10)__.

Part II. Calculation problems:

Note: You should carefully state the reasons or calculations in the following questions in order to get the points. A short answer, such as "Yes" or "No" will NOT receive any point.

1. (20%) Suppose you estimate the consumption function

$$Y_i = \alpha_1 + \alpha_2 X_i + u_{1,i}, \qquad i = 1, \dots, N$$

and the savings function

$$Z_i = \beta_1 + \beta_2 X_i + u_{2,i}$$

where $Y =$ consumption, $Z=$ savings, $X=$ income and $X = Y + Z$; that is, income is equal to consumption plus savings.

(1) What is the relationship, if any, between α_2 and β_2? Show your calculations. (5%)

(2) Will the residual sum of squares (RSS) be the same for these two models? Explain. (10%)

(3) Can you compare the coefficient of determinant R^2 of the two models ? Why or why not? (5%)

2. (15%) Consider the regression model

$$Y_i = \beta_1 + u_i, \quad i = 1, \dots, n.$$

where u_i satisfies all the standard assumptions for a linear regression and $\text{Var}(u_i) = \sigma^2$.

(1) Find the ordinary least squares (OLS) estimator $\hat{\beta}_1$ of β_1. (5%)

(2) Find $\text{Var}(\hat{\beta}_1)$. (10%)

3. (15%) Given the random sample (Y_i, X_i), where $i = 1, \ldots, n$, you use the OLS approach to estimate the following model

$$Y_i = \beta_2 X_i + e_i.$$

Assume the stochastic error e_i satisfies all the standard assumptions for a linear regression and $\mathrm{Var}(e_i) = \sigma^2$.

(1) Find the OLS estimator $\hat{\beta}_2$ of β_2. (5%)

(2) Find $\mathrm{Var}(\hat{\beta}_2)$. (5%)

(3) Suppose you want to test the null hypothesis H_0: $\beta_2 = 1$ and the alternative hypothesis H_a: $\beta_2 \neq 1$. State your test statistic, its distribution and the decision rule of the test given the level of significance α. (5%)